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II. *Recurrence Phenomena in Terrestrial Magnetism.*

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§ 1. The international scheme with headquarters at De Bilt, Netherlands, which supplies for each Greenwich day a magnetic character figure varying from 0·0 (very quiet) to 2·0 (very disturbed), has been in operation since January 1, 1906. The publication in 1926 of the results for 1925 brought up to 20 the number of complete years for which data are available. Of late, owing partly to a supposed connection between wireless and magnetic phenomena, the existence of a 27-day interval in magnetic disturbance has received increased attention. Further, Dr. Deslandres, of Meudon Observatory, has put forward the view that, in addition to the 27-day interval T, there are shorter intervals iT/6, where i is integral. Thus the time seemed to have arrived for carrying out a more exhaustive enquiry than was possible when the international character figures were first used to prove the existence of the 27-day interval. The length of the interval has been generally accepted as evidence that its ultimate cause is resident in the sun. In accordance with ideas prevalent since the time of the late Prof. Kr. Birkeland, it is supposed that magnetic disturbance is due to the discharge from the sun of some form of electricity carrier, and it is often assumed, following Birkeland, that sunspots are the areas where the discharge originates. During the 20 years 1906 to 1925, sunspot frequency and the mean annual solar latitude of spots have varied as shown in Table I.

Table I.—Sunspot Frequency and Mean Sunspot Latitude.

Year.	1906.	1907.	1908.	1909.	1910.	1911.	1912.	1913.	1914.	1915.
Sunspot frequency .	53·8	62·0	48·5	43·9	18·6	5·7	3·6	1·4	9·6	47·4
Mean sunspot latitude	13°·99	12°·12	10°·38	9°·71	10°·53	6°·49	8°·06	23°·23	21°·79	18°·77
Year.	**1916.**	**1917.**	**1918.**	**1919.**	**1920.**	**1921.**	**1922.**	**1923.**	**1924.**	**1925.**
Sunspot frequency .	57·1	103·9	80·6	63·6	37·6	26·1	14·2	5·8	16·7	44·3
Mean sunspot latitude	15°·81	14°·63	12°·75	10°·76	10°·43	7°·90	8°·02	15°·26	22°·73	20°·20

In our treatment of the subject, the 11 years 1906 to 1909, 1915 to 1920 and 1925 are regarded as years of many sunspots, the remaining 9 years as years of few sunspots. Again, 1906, 1907, 1913 to 1918, and 1923 to 1925 are regarded as years of high spot latitude, the remaining 9 years as years of low spot latitude. The reason for grouping the years was the large part played by "accident" in individual years.

As is well known, the rotation period of the sun, as shown by sunspots, increases with the solar latitude. The difference between the rotation periods for latitudes 10° and 20° is about half a day. If, then, sunspots are the only sources of the radiation causing magnetic disturbance on the earth, we should expect a very sensible difference between the recurrence periods in years of low and of high spot latitude.

The general method adopted was to form a "primary pulse" by entering under day n (or as it is more usually called, o) the international character figure for each of a group of days selected as representative of greater or less disturbance. The entries in column $n + 1$ (or 1) are the characters for the days immediately following the selected days, and so on. Starting with 1906, the international lists have given for each month 5 days selected as representative of the quietest conditions; these are the selected quiet (Q) days used here. It is only since 1923 that the lists have specified the 5 days considered the most disturbed of the month. For earlier years the 5 days (D days) chosen to represent disturbance are those having the largest character figures. Sometimes, for the last place or places in the list, a choice had to be made between two or more days with equal character figures. In making the choice no one principle was strictly adhered to, but no regard was ever paid to the character of the days which were 27 days earlier or later. As a matter of curiosity, it was subsequently investigated whether the choice made had in any way favoured the development of the 27-day interval. It was found that, if anything, the reverse was the case.

§ 2. Table II summarises the results of several investigations designed to throw light on the general features of disturbance. It seemed possible that the results obtained might depend on the number of days contributing to the primary pulse, or upon the choice of an invariable number from each month. The first group in Table II includes only those days in the 20 years 1906 to 1925 which had a character figure not less than 1·5, which represents really high disturbance. There were in all 365 of these days, or practically one day in twenty. The number in reality varied very much from year to year, being 31 in 1919, 30 in 1918 and 27 in 1917 (the year of sunspot maximum), but only 4 in 1912 and 1913, and 8 in 1914.

The mean character figure for all days of the 20 years was 0·619. The mean character figure from the 365 days of character $\nless 1·5$ was 1·694. The excess of this above the normal 0·619, viz., 1·075, is entered in column n of Table II as 1075, the unit employed in the table being 0·001 international character figure. If we weighted the 20 years according to the number of days which each contributed to the 365 primary days, the normal character would become 0·641, and it may be argued that this would have been a fairer level from which to measure both the primary pulse in the columns $n - 3$ to $n + 3$

Table II.—Results from 20 years 1906 to 1925. Departures from normal (unit 0·001 character unit).

Days forming primary pulse.	Primary pulse.							First secondary subsequent pulse.					
	$n-3$.	$n-2$.	$n-1$.	n.	$n+1$.	$n+2$.	$n+3$.	$n+25$.	$n+26$.	$n+27$.	$n+28$.	$n+29$.	$n+30$.
Days character $\nless 1\cdot5$ . . . . . . (365)	−27	28	406	1075	600	269	106	−16	84	199	202	156	87
3 largest of 5 D days . . . . . . (720)	−94	−40	305	820	419	141	12	−28	111	203	168	117	49
All 5 D days . . . . (1200)	−71	11	301	700	362	103	−14	17	114	182	144	84	40
D days character $< 1\cdot5$ . . . . (844)	−84	7	258	541	263	35	−64	30	124	170	115	53	15
2 least of 5 D days . . . . . . (480)	−37	87	295	521	277	45	−53	85	118	149	107	40	26
Days character 0·7 . . . . . . (476)	42	25	50	81	−36	−44	−3	37	16	1	−51	−69	−12
Days characters 0·7, 0·6 and 0·5 (1369)	27	21	3	−16	−67	−60	−19	8	−11	−24	−49	−40	−18
Days character 0·6 . . . . . . (444)	31	29	−8	−19	−39	−52	−37	3	−7	−25	−49	−33	−23
Days character 0·5 . . . . . . (449)	8	9	−34	−119	−128	−85	−19	−16	−43	−50	−48	−16	−19
All 5 Q days . . . . (1200)	5	−103	−304	−527	−224	14	67	−73	−126	−149	−90	−29	4

and the secondary (subsequent) pulse in the columns $n + 25$ to $n + 30$. The difference this would cause in the primary pulse would be trifling, but it would reduce the amplitude of the secondary pulse very sensibly.

The 20 years supplied 1,200 ($5 \times 12 \times 20$) selected D days; the results from these and their associated days appear in the third line of Table II. Of the 5 D days of each month the 3 with highest character figures, 720 in all, and their associated days supplied the data in the second line of the table, and the balance of the D days—*i.e.*, the 2 of the 5 D days having the lowest character figures—numbering 480 and their associated days made up the 5th line. The 4th line includes those of the selected D days, 844 in number, which had character figures less than 1·5. The excess 9 in the combined totals of the days forming the primary pulses in the 1st and 4th lines of the table over 1,200, the number of selected D days, comes from months when more than 5 days had characters of 1·5 or more; two of the extra days came from August, 1917, and two from May, 1921.

The primary pulses in the first five lines of Table II represent a gradually diminishing intensity of disturbance, which is recognisable in the corresponding five curves AA to EE in fig. 1. The height of the peak on day n (called "o" in the figure) above the normal line—which represents 0·619 in each case—becomes gradually less. In the case of AA the character on day $n - 3$ falls below, while that on day $n + 3$ is still much above the normal; also the character figures for days $n + 1$ and $n + 2$ respectively notably exceed those for days $n - 1$ and $n - 2$. There is thus a marked skewness in AA. Skewness in the same direction, but gradually diminishing, is also seen in curves BB, CC and DD. But in DD it is very slight, and in EE it has changed its type, the character figures for the days subsequent to the crest being now a shade less than those for the corresponding days preceding it. The fact that, in these five cases, the last three of the character figures for days $n + 3$ and all the character figures for days $n - 3$ fall below the normal is not really strange. We have collected in column n the outstanding disturbed days of the month, and this tends to reduce below the normal the character figures in all the other columns. The mean character figures for days $n - 1$, $n + 1$ and $n + 2$ exceed the normal only because of the tendency for disturbed and quiet days to occur in groups. This fact has to be remembered, even in connection with the secondary pulse. A selected D day has only 4 chances as against the 5 for the non-selected days of contributing to the secondary pulse in its own month, and its chance of contributing is further reduced when the selected D days occur in groups. But even in a 31-day month only the first four days have the 27th subsequent day in the same month. Except in a wholly exceptional case the great majority of the days contributing to the secondary (subsequent) pulse belong to the following month, and the *direct* influence on the secondary pulse of the choice of days made for the primary pulse is insignificant.

In short, unless a real tendency to recurrence after a 27-day interval existed, the secondary pulse, apart from accident, would be only of the most insignificant amplitude, and opposite in sign to the primary pulse. The difference in the amplitudes of the

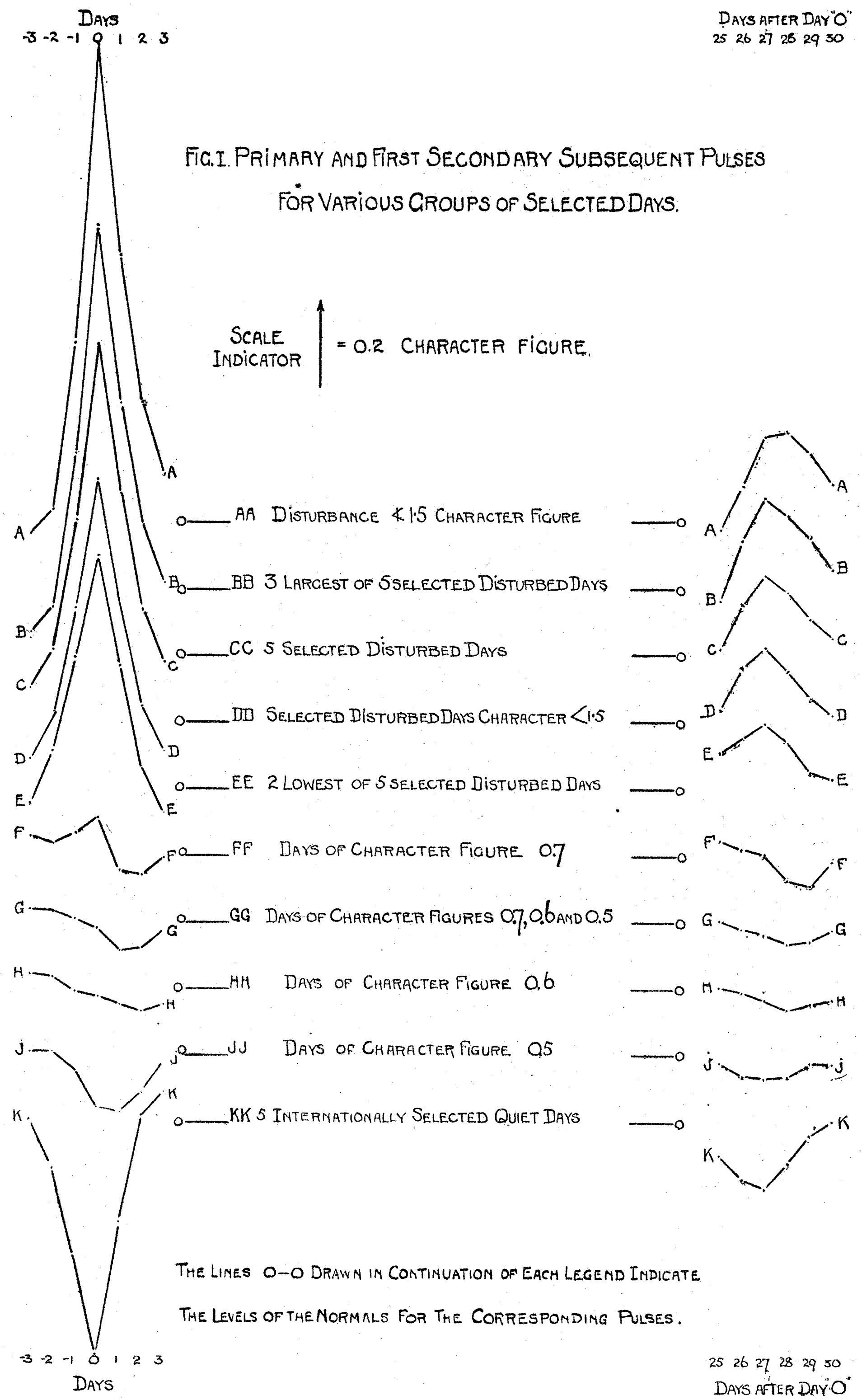
Days
-3 -2 -1 0 1 2 3
Days after Day "O"
25 26 27 28 29 30
Fig. 1. Primary and First Secondary Subsequent Pulses
For Various Groups of Selected Days.
Scale Indicator
= 0.2 Character Figure.
AA Disturbance ≮ 1·5 Character Figure
BB 3 Largest of 5 Selected Disturbed Days
CC 5 Selected Disturbed Days
DD Selected Disturbed Days Character < 1·5
EE 2 Lowest of 5 Selected Disturbed Days
FF Days of Character Figure 0.7
GG Days of Character Figures 0.7, 0.6 and 0.5
HH Days of Character Figure 0.6
JJ Days of Character Figure 0.5
KK 5 Internationally Selected Quiet Days
The Lines O—O Drawn in Continuation of Each Legend Indicate
The Levels of the Normals for the Corresponding Pulses.
-3 -2 -1 0 1 2 3
Days
25 26 27 28 29 30
Days after Day "O"

Fig. 1.

secondary pulses answering to the five primary pulses AA to EE in fig. 1 is sensible, but makes less appeal to the eye than the gradual change of skewness. In the case of the secondary pulses AA, BB and CC, exactly as in the case of the corresponding primary pulses, the ordinate for the day two days subsequent to the day $n + 27$, regarded as the crest, is in excess of that for the day two days previous to the crest. The difference between these pairs of ordinates gradually diminishes, changes sign as we pass from CC to DD, and in EE is decidedly the other way. This skewness is a source of trouble when it comes to deciding exactly how long the interval is.

§ 3. Lines 6 to 9 of Table II and curves FF, GG, HH and JJ of fig. 1 represent an attempt to find whether characteristics intermediate between the disturbed and the quiet show anything answering to a 27-day interval. The mean character figure for the 20 years being 0·619, the 476 days of character 0·7 which contribute to the primary pulse FF, and to line 6 in the table, represent sensibly more than average disturbance. The 444 days of character 0·6 contributing to the primary pulse HH, and the 449 days of character 0·5 contributing to the primary pulse JJ, represent in the one case a very slight, and in the other case a very sensible, fall below normal disturbance.

The slant down from left to right is one of the most prominent features in the primary pulses FF and HH, and it also presents itself in the corresponding secondary pulses.

The primary pulse FF shows, in addition to the peak in day n, a hollow in day $n + 2$. The peak is represented in the secondary pulse only by an arrest in the downward slope on day $n + 27$, but the hollow is fairly represented at day $n + 29$. The departure from a straight line in the primary pulse HH is trifling; in the secondary pulse the tendency to a concavity with turning point at day $n + 28$ is more apparent. In the case of JJ, the concavity is quite prominent in both the primary and secondary pulses, but the minimum appears on day $n + 1$.

The special features in the primary pulses FF and HH, and to a minor extent in JJ, arise from the fact that character figures 0·7, 0·6 and 0·5, especially the first two, occur most often when disturbance is on the down grade. As is suggested by the shape of the primary pulses AA to CC, the rise to disturbance, especially to outstandingly large disturbance, is usually more rapid than the decline. The days of characters 0·7, 0·6 and 0·5, when combined, as in the 7th line of Table II, and in the curve GG of fig. 1, supply a fairly regular primary pulse, and the corresponding secondary pulse is similarly regular. The minima in these pulses appear on days $n + 1$ and $n + 28$. The 27-day interval is thus clearly apparent, the only special feature being the relatively large amplitude of the secondary pulse.

The last line in Table II and the curve KK of fig. 1 are based on the international quiet and associated days. The recovery from the depressions at days n and $n + 27$ is more rapid than the previous falls.

The exhibition of a 27-day interval in groups of days of all types, from the most highly disturbed to the quietest, seems to imply that there is no exceptional phenomenon on highly disturbed days, but merely increase in the activity of some agent always more or less active. If magnetic disturbance is due to radiation from the sun, then, unless the effects of the radiation persist for some days, the radiation must always be going on ; it is merely a case of more or less. If the radiation received by the earth on a given day proceeds from a definite limited sector or area on the sun, then a sector which is pre-eminent for its radiation at a given date has a better chance than the average sector of being pre-eminent after 27 days have elapsed.

§ 4. Tables III to VII are constructed on similar lines to Table II, but each includes 4 secondary pulses. In the case of Table II, the days in the secondary pulses answering to the primary D and Q days of December, 1925, belonged mainly to January, 1926. In the absence of official character figures, provisional character figures*—which could not have been in error by more than 0·1—were calculated from the data for the individual co-operating observatories contained in the De Bilt return for the first quarter of 1926. But when it came to the second, third and fourth secondary pulses, the employment of data from 1925 in the primary pulse would have called for a considerable number of provisional data for 1926. This seemed undesirable. Consequently, Tables III and IV, which refer to subsequent secondary pulses, contain primary pulses which depend entirely on the 19 years 1906 to 1924. This entails slight differences between corresponding figures in these tables and in Table II. Table III aims partly at showing what influences the varying amplitude of the primary pulse exerts on the amplitude of the secondary pulses, but its main object and that of Tables IV to VII is to ascertain what differences exist between the contrasted groups of years, between years of low and high spot latitude in the one case, and between years of few and many sunspots in the other.

Tables V and VI deal with the secondary *previous* pulses, *i.e.*, with the pulses having their maxima, or minima, approximately 27, 54, 81 and 108 days *earlier* than the primary pulse. In their case information was entirely lacking as to the associated days answering to the days in the primary pulse from the earlier months of 1906. It was decided to omit 1906 from the primary pulses, deriving them entirely from the 19 years 1907 to 1925. The years of low spot latitude and the years of few sunspots, nine in either group, were the same in all cases. But in Tables III and IV 1906 was included amongst the years of high spot latitude and of many sunspots, while in Tables V and VI it was replaced by 1925. The consequence is a small difference between the primary pulses in Tables III and V and in Tables IV and VI for the 19 years, and for the groups of years of high spot latitude and of many spots.

In Tables III and V, which deal with disturbed and associated days, the pulses are positive, *i.e.*, the figures represent excesses above the normal. In them a negative sign—*e.g.*, in the first column of Table III—signifies a deficiency in the character figure.

In Tables IV and VI, dealing with the international quiet and associated days, the

* July 8, 1927.—Of the 31 provisional characters for January, 1926, 23 agreed exactly with the final international figures, and 8 differed by 0·1.

Table III.—Positive Primary and Subsequent Pulses, 19 Years 1906 to 1924 (unit 0·001 character figure).

Days forming primary pulse.	Primary pulse.							1st secondary pulse.					
	—3.	—2.	—1.	0.	1.	2.	3.	25.	26.	27.	28.	29.	30.
Days of character $\nless 1\cdot5$ (339)	—30	34	411	1074	601	275	99	—15	92	207	213	161	79
3 largest of 5 D days . . (684)	—93	—35	307	815	420	144	11	—30	110	208	169	112	46
All 5 D days . . . . . (1140)	—71	16	302	696	365	108	—15	17	116	185	146	88	39
D days character $< 1\cdot5$ . .(808)	—83	13	258	540	269	40	—61	29	124	174	116	55	18
2 least of 5 D days . . . (456)	—39	93	295	518	282	52	—53	87	125	152	112	53	28
All selected days from—													
9 years of low spot latitude . .	—88	12	287	679	364	113	—13	4	128	229	178	92	26
10 years of high spot latitude .	—57	19	314	710	365	102	—17	28	105	145	117	84	49
9 years of few sunspots . . .	—52	37	296	681	358	124	9	25	149	227	170	115	53
10 years of many sunspots . .	—89	— 3	307	709	371	92	—36	9	86	148	124	63	25

Days forming primary pulse.	2nd secondary pulse.					3rd secondary pulse.					4th secondary pulse.				
	52.	53.	54.	55.	56.	79.	80.	81.	82.	83.	106.	107.	108.	109.	110.
Days of character $\nless 1\cdot5$. (339)	27	68	141	155	115	—29	19	61	58	42	12	19	71	56	29
3 largest of 5 D days . . . (684)	21	71	123	130	67	—10	34	55	41	11	25	28	48	40	10
All 5 D days . . . . . .(1140)	22	76	121	100	53	17	43	52	33	12	7	20	29	26	3
D days character $< 1\cdot5$. (808)	18	79	112	76	26	36	52	45	21	— 2	5	20	12	13	— 9
2 least of 5 D days . . . (456)	22	82	117	56	32	43	57	47	20	13	—21	9	2	6	— 8
All selected days from—															
9 years of low spot latitude . .	14	82	146	120	63	4	25	66	67	41	—14	14	36	37	—12
10 days of high spot latitude . .	28	69	98	82	43	27	59	38	1	—15	24	25	22	16	15
9 years of few sunspots	6	102	165	137	67	4	56	100	72	24	3	40	40	17	—25
10 years of many sunspots . .	35	51	81	67	40	27	31	8	— 3	0	10	2	19	35	27

TABLE IV.—Negative Primary and Subsequent Pulses, 19 Years 1906 to 1924 (unit 0·001 character figure).

Groups of years.	Primary pulse.							1st secondary pulse.					
	−3.	−2.	−1.	0.	1.	2.	3.	25.	26.	27.	28.	29.	30.
19 years 1906–1924	+10	100	304	526	223	+17	+69	73	125	149	90	27	+ 3
9 years of low spot latitude	0	90	291	531	216	+47	+93	83	133	150	89	25	+12
10 years of high spot latitude	+18	110	316	522	230	10	+49	66	119	149	92	30	5
9 years of few sunspots	+14	97	279	490	199	+29	+65	93	146	156	95	1	+20
10 years of many sunspots	+ 5	102	326	559	245	+ 6	+73	57	107	144	86	51	12

Groups of years.	2nd secondary pulse.					3rd secondary pulse.					4th secondary pulse.				
	52.	53.	54.	55.	56.	79.	80.	81.	82.	83.	106.	107.	108.	109.	110.
19 years 1906–1924	56	92	89	48	11	31	45	17	+ 1	1	9	25	35	33	27
9 years of low spot latitude	61	83	97	42	4	41	69	27	8	32	18	51	43	45	36
10 years of high spot latitude	52	100	83	55	18	24	24	9	+ 9	+26	3	3	28	23	19
9 years of few sunspots	55	103	96	45	+ 7	41	50	16	14	+20	5	28	31	35	15
10 years of many sunspots	57	81	83	51	28	23	41	19	+14	20	14	23	38	32	38

TABLE V.—Positive Primary and Previous Pulses, 19 Years 1907 to 1925 (unit 0·001 character figure).

Group of years.	4th secondary pulse.					3rd secondary pulse.					2nd secondary pulse.				
	—110.	—109.	—108.	—107.	—106.	—83.	—82.	—81.	—80.	—79.	—56.	—55.	—54.	—53.	—52.
19 years 1907–1925	— 2	17	40	34	4	1	16	41	61	34	27	84	120	87	47
9 years of low spot latitude	18	54	64	59	9	30	41	56	70	50	58	89	119	102	77
10 years of high spot latitude	—20	—16	20	12	— 1	—25	— 5	27	54	20	1	80	122	74	21
9 years of few sunspots	—23	26	58	37	11	—16	43	78	84	34	44	100	136	109	50
10 years of many sunspots	16	8	25	31	— 2	16	— 8	7	40	34	13	69	105	67	45

Group of years.	1st secondary pulse.					Primary pulse.						
	—29.	—28.	—27.	—26.	—25.	—3.	—2.	—1.	0.	1.	2.	3.
19 years 1907–1925	42	117	173	152	89	—71	11	303	703	363	109	— 9
9 years of low spot latitude	24	120	191	168	105	—88	12	287	679	364	113	—13
10 years of high spot latitude	59	116	157	138	75	—55	11	317	725	363	107	— 5
9 years of few sunspots	64	148	204	194	106	—52	37	296	681	358	124	9
10 years of many sunspots	23	90	145	114	74	—88	—12	309	723	368	96	—25

TABLE VI.—Negative Primary and Previous Pulses, 19 Years 1907 to 1925 (unit 0·001 character figure).

Group of years.	4th secondary pulse.					3rd secondary pulse.					2nd secondary pulse.				
	−110.	−109.	−108.	−107.	−106.	−83.	−82.	−81.	−80.	−79.	−56.	−55.	−54.	−53.	−52.
19 years 1907–1925	19	27	33	35	21	+ 6	30	37	28	17	61	111	75	50	13
9 years of low spot latitude	+ 8	4	8	8	7	+14	21	37	18	2	28	103	84	43	3
10 years of high spot latitude	42	46	55	60	32	0	37	37	36	31	91	117	67	55	21
9 years of few sunspots	+ 9	2	6	29	21	+28	33	50	23	17	31	100	68	41	+ 9
10 years of many sunspots	43	48	56	41	20	14	27	26	32	18	88	120	81	58	33

Group of years.	1st secondary pulse.					Primary pulse.						
	−29.	−28.	−27.	−26.	−25.	−3.	−2.	−1.	0.	1.	2.	3.
19 years 1907–1925	66	102	126	81	20	+ 4	101	303	528	221	+22	+69
9 years of low spot latitude	73	107	147	90	+ 1	0	90	291	531	216	+47	+93
10 years of high spot latitude	58	97	107	73	38	+ 9	111	313	523	225	0	+50
9 years of few sunspots	59	108	140	79	22	+14	97	279	490	199	+29	+65
10 years of many sunspots	72	97	114	83	18	4	105	325	561	241	+15	+73

TABLE VII.—Difference Pulses, Subsequent and Previous (reversed) (unit 0·001 character figure).

Group of years.	Primary pulse.							1st secondary pulse.					
	—3.	—2.	—1.	0.	1.	2.	3.	25.	26.	27.	28.	29.	30.
All years	—159	203	1190	2453	1194	203	—159	199	474	633	455	223	50
Years of low spot latitude	—194	168	1158	2420	1158	168	—194	191	519	717	494	214	24
Years of high spot latitude	—130	236	1218	2480	1225	234	—130	207	435	558	422	231	70
Years of few sunspots	—122	229	1132	2342	1132	229	—122	246	568	727	521	239	37
Years of many sunspots	—192	180	1242	2552	1250	179	—193	158	390	551	397	209	60

Group of years.	2nd secondary pulse.					3rd secondary pulse.					4th secondary pulse.				
	52.	53.	54.	55.	56.	79.	80.	81.	82.	83.	106.	107.	108.	109.	110.
All years	138	305	405	343	152	99	177	147	78	8	41	114	137	103	47
Years of low spot latitude	155	310	446	354	153	97	182	186	137	89	20	132	151	140	34
Years of high spot latitude	122	298	370	334	153	102	173	111	24	—66	58	100	125	69	56
Years of few sunspots	102	355	465	382	135	96	213	244	162	—40	40	134	135	80	—42
Years of many sunspots	170	257	350	307	169	102	144	60	2	50	42	97	138	123	124

pulses are negative, the figures representing depression below the normal for the group of years. But when a + sign appears, as on day 3 of Table IV, the normal was exceeded. The normal values were 0·621 for the 19 years 1906 to 1924, 0·617 for the 19 years 1907 to 1925, 0·634 for the nine years of low spot latitude, 0·568 for the nine years of few sunspots, 0·610 in Tables III and IV and 0·601 in Tables V and VI for the 10 years of high spot latitude, and, finally, 0·669 in Tables III and IV and 0·661 in Tables V and VI for the years of many sunspots.

Difference pulses, not reproduced in a Table, were calculated from Tables III and IV, and from Tables V and VI. A "difference pulse" represents the algebraic difference between the corresponding figures in positive and negative pulses. For example, the representative D day contributing to day 0 in Table III had a character figure greater by 0·696 than the normal, 0·621, for the 19 years, while the representative Q day contributing to day 0 in Table IV had a character figure less by 0·526 than the same normal. Thus, taking 0·001 character figure for our unit, we get 696 + 526, or 1222, as the entry for day 0 in the primary difference pulse. Similarly for day + 27 of the corresponding secondary difference pulse we should get 185 + 149, or 334. The corresponding figures for the 19 years previous difference pulses derived from Tables V and VI are 1231 (703 + 528) for day 0, and 299 (173 + 126) for day — 27.

All the primary pulses derived direct from Tables III to VI, whether they be positive, negative or difference pulses, have perceptible skewness. As this introduces uncertainty into the length of the interval, it was sought to obtain symmetrical primary pulses. This was accomplished by counting time backwards in the case of the previous pulses, and combining them thus reversed with the subsequent pulses. Day + 3 was regarded as the start of the primary previous pulse, and day — 110 as the end of its fourth secondary pulse. The results thus obtained from the difference pulses based on Tables III to VI appear in Table VII. The primary pulses in Table VII are necessarily completely symmetrical for the groups of years of low spot latitude and few sunspots.

In the remaining three cases one year was different as between the subsequent and previous primary pulses, but the departures from symmetry in the resulting primary pulses in the 1st, 3rd and 5th lines of Table VII are so trifling that any error entailed in accepting O as the centre of the primary pulse must be exceedingly small. It is on Table VII we shall mainly rely when estimating the length of the interval.

§ 5. Figs. 2 to 4 show some of the results diagrammatically. Fig. 2, representing the results from all the years, corresponds to a figure in an earlier paper,* which dealt with the 6 years 1906 to 1911, but it includes an additional pair of pulses. Even 20 years do not supply results of perfect smoothness, but it is fairly obvious that we could, if we chose, make out secondary pulses beyond the fourth. It may be well to point out that the increase from left to right in the amplitude of the previous secondary pulses, and the corresponding decrease in the subsequent pulses, represent not a waxing and

* 'Phil. Trans.,' A, vol. 213, p. 253.

waning in the amplitude of disturbance but the lesser probability of two recurrences than of one, of three recurrences than of two, and so on. If we suppose a disturbance recurring in 27 days of sufficient intensity to figure as a D day on five successive occasions, it will contribute every time to the primary pulse. The first occurrence contributes to all the previous pulses, the second to the pulses centering at — 81, — 54, — 27 and + 27 days, the third to the pulses centering at — 54, — 27, + 27 and + 54 days, the fourth to the pulses centering at — 27, + 27, + 54 and + 81 days, and the fifth only to the subsequent pulses. Supposing the character figure to decline steadily, then each previous pulse would receive a larger contribution than the corresponding subsequent pulse. A tendency to wane would show itself in a diminished amplitude of each subsequent positive pulse as compared with the amplitude of the corresponding previous positive

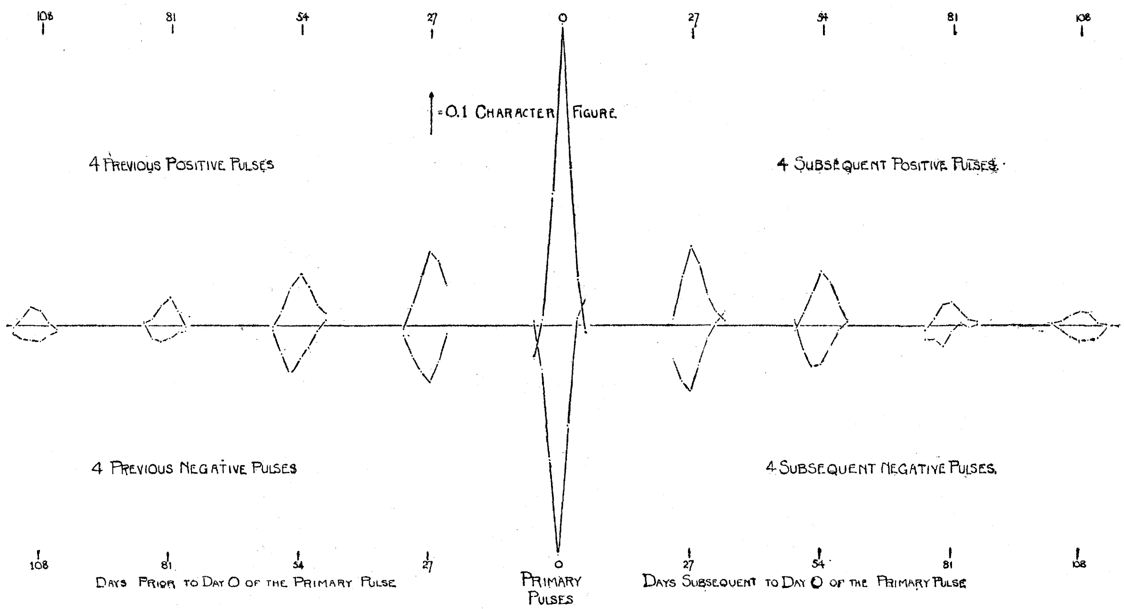

Fig. 2.—Recurrences of Positive and Negative Pulses.

pulse. If we take the all-year results represented in fig. 2 as being the least exposed to accidental features, we see no decided tendency for the subsequent pulses to be either less or greater than the previous. If the positive pulse centering at + 27 days slightly exceeds that centering at — 27 days, the positive pulse centering at + 108 days is less than that centering at — 108 days.

Fig. 2 shows that, relative to their primary pulse, the secondary negative pulses are quite as prominent as the secondary positive pulses.

§ 6. The main object in investigating so many secondary pulses was to obtain a more exact measure of the length of the interval. Supposing, for example, the interval in the average year had been 27·25 days, then, going to the nearest whole day, we should have expected to find crests at ± 27, ± 54 or 55, ± 82 and ± 109 days. Taking the

5 selected D days for 19 years, what we really find in the positive pulses are crests at $\pm$ 27 days, $\pm$ 54 days, — 80 days, + 81 days and $\pm$ 108 days; while the corresponding negative pulses answering to the selected Q days have minima at $\pm$ 27 days, + 53 days, — 55 days, + 80 days, — 81 days, — 107 days and + 108 days. The excess of the negative value at — 107 days over that at — 108 days is no greater than the excess of the negative value at + 108 days over that at + 109 days. In short, so far, as the first, second and fourth secondary pulses are concerned, we should infer that any difference in the length of the interval from 27 days must be very small. But the third secondary pulses, both positive and negative, do suggest a somewhat shorter interval.

It was largely with a view to investigating this point more closely that Table VII was constructed. The results, so far as the 19 years are concerned, are shown graphically in fig. 3. To bring out more clearly resemblances or differences in type, the departures

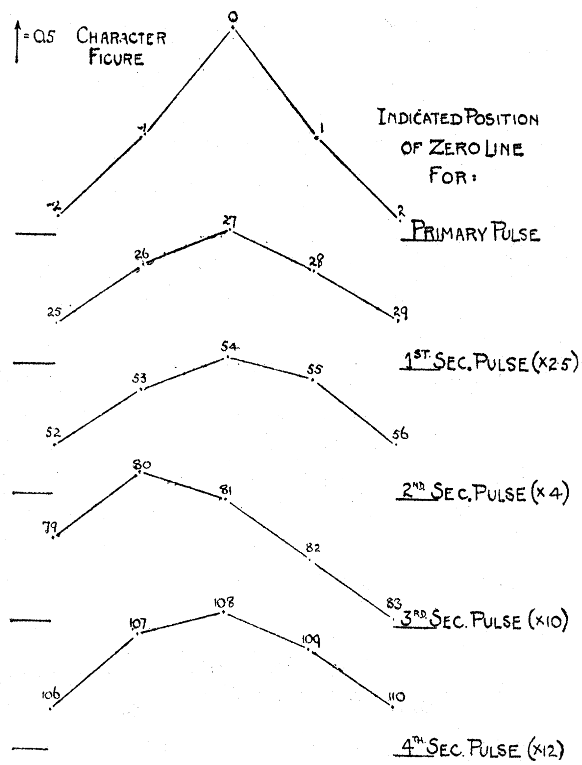

Fig. 3.—Difference Pulses—Subsequent (1906–24) and previous (1907–25) reversed. The numbers indicate the day represented by the point as related to the day 0 of the primary pulse.

from the normal which contribute to the ordinates in fig. 3 have had convenient multipliers applied, viz., 2·5 for the first secondary pulse, 4 for the second, 10 for the third and 12 for the fourth. The ordinates for days, 0, 27, 54, 81 and 108 are placed in a line, one below the other, each pulse having a zero line of its own. The primary pulse is, for the reasons already stated, practically symmetrical, and closely resembles two straight lines. The first, second and fourth secondary pulses differ from the primary, all showing a less concentration of the pulse towards its centre, but they are very similar to one another. From their appearance one would hesitate to draw any conclusion as to the direction in which the fundamental interval differs from 27 days. But the third secondary pulse differs from the others and throws the maximum well on the down side of 81 days. It may be pointed out in passing that to bring the size of the third secondary pulse up to that of the others, we had to apply a multiplier which is high as compared with the multipliers applied to the second and fourth pulses.

§ 7. Fig. 4, in essentials analogous to fig. 3, illustrates the results obtained in Table VII for the contrasted groups of years. It is only in the case of the third secondary

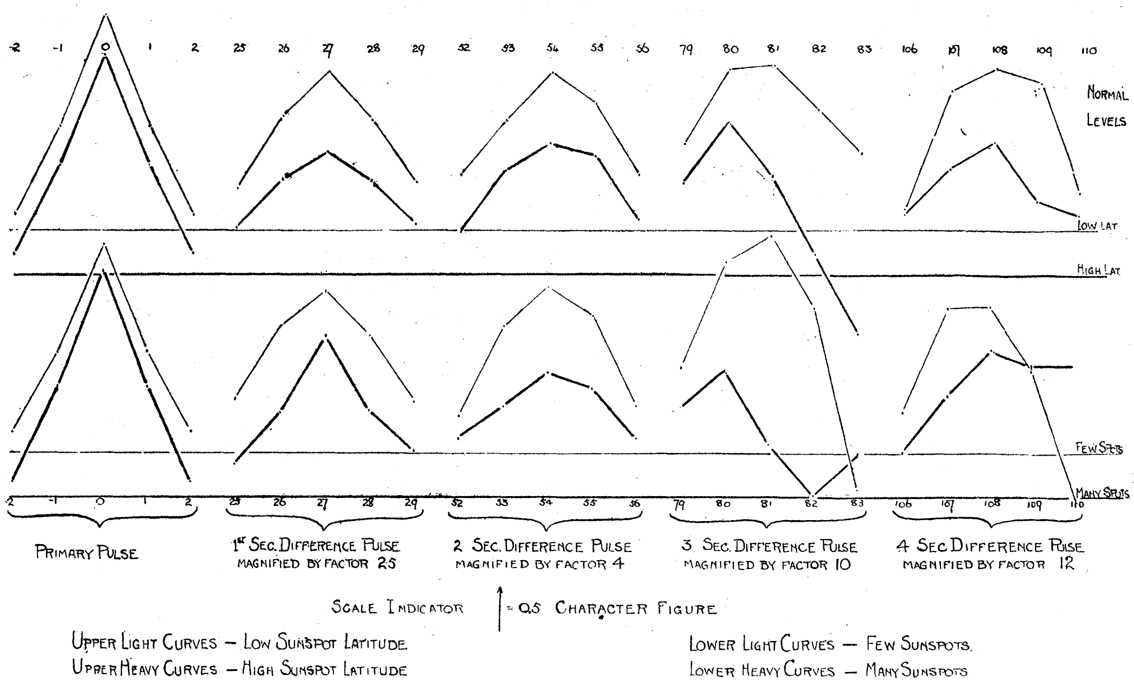

Fig. 4.—Different Pulses (Subsequent and Previous Reversed) for Various Groups of Years.

pulse that any difference appears in the interval, and in that case it is the high spot latitude years that supply the shorter interval ! The maximum falls on day 81 in the case of the low spot latitude years, but the excess on day 81 over day 80 is hardly convincing. Thus, while a difference appears between the years of low and high spot latitude, in the direction *opposite* to expectations, it is small, only a fraction of a day. The difference, such as it is, tends to be discounted by the fact that the fourth secondary pulses from both groups of years have their crests on day 108.

It is the group of years of few sunspots that gives a crest most clearly on day 81, and the third secondary pulse is much better developed in this case than in any other. The group of years of many sunspots give very irregular results for both the third and fourth secondary pulses.

As Table VII shows, there is no great difference between the amplitudes of the primary pulses for the years of low and high spot latitude, but the latter is rather the larger. In the secondary pulses, however, the excess lies decisively with the years of low spot latitude. As between the years of few and many sunspots, the latter supply decidedly the larger primary pulses, but the excess of the secondary pulses from the years of few sunspots is quite large for the first three, especially for the third. The fourth secondary pulse is, however, rather an exception. The calculations for the secondary pulses were really made for several days beyond the limits of the tables, and it appeared that the deficiency in the secondary pulses for the years of many sunspots, as compared with the years of few spots, was due in considerable measure to a greater spreading of the pulses in time. The fourth subsequent positive pulse for the years of few sunspots had positive ordinates only from day 106 to day 109, while the corresponding pulse for the years of many sunspots had positive ordinates right from day 104 to day 112.

§ 8. Table VIII shows the results of a more distinctly mathematical calculation of the position of the crests of the several pulses, and the interval between each secondary crest and the corresponding primary crest. Use was made in each case of the mean character figures for 5 successive days, the central day being ± 27, ± 54, ± 81 or ± 108. Take, for example, the primary positive pulse for the 19 years 1906 to 1924. The result $+0{\cdot}17$, given under 0 in Table VIII, is the position of the centre of gravity of weights 16 at -2 (days), 302 at -1, 696 at 0, 365 at $+1$, and 108 at $+2$ (see Table III). If we deal similarly with the results from days $+25$ to $+29$, we find the crest of the first secondary pulse at $27{\cdot}31$ days. From these two results we obtain for the length of the interval $27{\cdot}31-0{\cdot}17$, or $27{\cdot}14$ days, and the excess $+0{\cdot}14$ above 27 days appears in the first of the columns headed "Lengths of Intervals." In this case, going to $0{\cdot}01$ day only, we get the same position, $+0{\cdot}17$, for the crest of the primary pulse from the 19 years 1907 to 1925, answering to the previous secondary pulses. For the position of the crest of the corresponding first previous secondary pulse we get $-27+0{\cdot}22$ days. The calculated length of the interval is thus $+0{\cdot}17-(-27+0{\cdot}22)$ or $27-0{\cdot}05$ days. The departure $-0{\cdot}05$ from 27 days appears in the table.

The method was adopted with a view to eliminating, so far as possible, the effect of skewness in the primary pulses. It was desirable to ascertain what measure of agreement there really was between the positive and negative pulses, and between the subsequent and the previous pulses. Obviously, an odd number of days had to be employed, and 5 seemed distinctly preferable to 3. We suppose the results from the difference pulses to have smaller probable errors than the results from either the positive or the negative pulses separately, and the results from the difference pulses subsequent combined with previous reversed to be the most exact of the lot. But the increasing complexity and, some may think, artificiality of the latter pulses may appear an objection.

It will be observed that the crests of all the primary difference pulses fall on the

TABLE VIII.—Positions of Crests (Centres of Gravity) of Pulses and Lengths of Intervals.

Nature of primary pulses.	Position of crests.					Lengths of intervals.			
	0.	±27.	±54.	±81.	±108.	27.	54.	81.	108.
5 selected D or Q days a month.									
19 years—									
Positive subsequent pulses	+0·17	+0·31	+0·23	−0·13	−0·02	+0·14	+0·06	−0·30	−0·19
Negative subsequent pulses	−0·28	−0·27	−0·45	−1·14	+0·34	+0·01	−0·17	−0·86	+0·62
Positive previous pulses	+0·17	+0·22	+0·12	+0·72	+0·31	−0·05	+0·05	−0·55	−0·14
Negative previous pulses	−0·29	−0·29	−0·51	+0·41	+0·09	−0·00	+0·22	−0·70	−0·38
Subsequent difference pulses—									
19 years	−0·03	+0·04	−0·07	−0·50	+0·20	+0·07	−0·04	−0·47	+0·23
Years of low spot latitude	−0·03	+0·06	−0·03	+0·10	+0·22	+0·09	0·00	+0·13	+0·25
Years of high spot latitude	−0·03	+0·03	−0·11	−1·77	+0·14	+0·06	−0·08	−1·74	+0·17
Years of few sunspots	−0·04	−0·03	−0·03	−0·29	−0·28	+0·01	+0·01	−0·25	−0·24
Years of many sunspots	−0·02	+0·13	−0·11	−0·98	+0·52	+0·15	−0·09	−0·96	+0·54
Previous difference pulses—									
19 years	−0·03	+0·02	−0·17	+0·60	+0·26	−0·05	+0·14	−0·63	−0·29
Years of low spot latitude	−0·03	+0·04	−0·08	+0·31	+0·09	−0·07	+0·05	−0·34	−0·12
Years of high spot latitude	−0·03	−0·01	−0·26	+0·99	+0·26	−0·02	+0·23	−1·02	−0·29
Years of few sunspots	−0·04	+0·02	−0·18	+0·69	+1·05	−0·06	+0·14	−0·73	−1·09
Years of many sunspots	−0·02	0·00	−0·16	+0·47	−0·23	−0·02	+0·14	−0·49	+0·21
Difference pulses, subsequent and previous (reversed)—									
19 years						+0·01	+0·05	−0·55	0·00
Years of low spot latitude						+0·01	+0·03	−0·09	+0·07
Years of high spot latitude						+0·02	+0·08	−1·41	+0·09
Years of few sunspots						−0·03	+0·06	−0·48	−0·63
Years of many sunspots						+0·06	+0·04	−0·69	+0·36
Positive pulses from 20 years.									
Subsequent—									
Character not less than 1·5	+0·28	+0·74				+0·46			
3 largest of 5 D	+0·29	+0·58				+0·29			
D character less than 1·5	+0·05	+0·07				+0·02			
2 least of 5 D	−0·08	−0·20				−0·12			
Previous—Character not less than 1·5	+0·28	+0·34				−0·06			
Positive pulses from 19 years.									
Subsequent—Character not less than 1·5	+0·28	+0·72	+0·52	+1·20	+0·38	+0·44	+0·24	+0·92	+0·10

negative side of 0, but their departures from 0 are trifling as compared with the departures in the case of the primary positive and negative pulses. Also the differences between the different groups of years are insignificant. Coming to the secondary pulses, if we except the third, the intervals, calculated from the years of low and high spot latitude, differ but little, all closely approaching 27 days or multiples thereof. In the case of the subsequent difference pulses the low spot latitude years give in each instance the larger value of the interval. In the case of the other difference pulses the high spot latitude years give the longer interval from the first and second secondary pulses, but the shorter interval from the fourth secondary pulse. It is really only in the case of the third secondary pulse that Table VIII suggests any appreciable difference between years of low and high spot latitude. In that case the high spot latitude years give the shorter value for the interval. The difference appears greater for the subsequent than for the previous pulses. If we except the low spot latitude group of years, every case, whether of positive, negative or difference pulses makes the interval from the primary to the third secondary pulse less than 81 days.

In the case of the combined subsequent and previous pulses, the departures of the primary crests from 0 are negligible, and it suffices to give the lengths calculated for the intervals.

The departures from 27 and from 54 days are all very trifling, and in the case of the 19 years the departure from 108 days is the least of any. The fact that the departure from 81 days has the same sign for the group of years of few sunspots as for the others may appear inconsistent with what was said in discussing fig. 3. It is due to the relatively low values of the characters for days 82 and 83 as compared with days 80 and 79 respectively. Taking single days, we get the crest at day 81, but taking a group of days, whether 5 or 3, we get the crest decidedly on the down side of 81.

§ 9. The last six lines of Table VIII were suggested by the question whether the length of the interval is influenced by the amplitude of the primary pulse. Combining these 20-year data for the first subsequent pulse with the selected D and Q day data at the top of the table, we find, as we pass from the highest to the lowest intensity of the primary pulse, the following as the positions of the secondary crest: $27 + 0 \cdot 74$, $27 + 0 \cdot 58$, $27 + 0 \cdot 31$, $27 + 0 \cdot 07$, $27 - 0 \cdot 20$ and $27 - 0 \cdot 27$.

Thus if, disregarding the skewness of the primary pulse, we took $0 \cdot 00$ day as in each case the commencement of the interval, we should get a very perceptible shortening of the interval as disturbance diminished in the primary pulse. Even when we make allowance for the skewness of the primary pulse in the way adopted here, we do not wholly get rid of the phenomenon, but the evidence in its favour does not appear conclusive. Our final results for the length of the interval from the two lowest grades of disturbed days ought, if the phenomenon were true, to be decidedly longer than the length derived from the selected Q days; but the reverse is the case. There ought also to be a decided difference between the lengths of the intervals derived from the positive previous pulses for all D days and for the D days of characters not less than $1 \cdot 5$.

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The difference is insignificant and is in the wrong direction. The fact that so big a difference exists in the case of the most highly disturbed group of days between the lengths of the intervals, as derived from the subsequent and previous pulses, raises a doubt as to the complete success of the method we have adopted for eliminating the effect of skewness when the skewness is large. If we took as the length of the interval half the interval between the crests of the first secondary pulses subsequent and previous, which closely resemble one another, we should find 27·20 from the days of character ≮ 1·5, as compared with 27·04 from the 5 selected D days, and 27·01 from the selected Q days. The intervals derived from the second, third and fourth subsequent pulses appear all slightly longer for the days of character ≮ 1·5 than for the 5 selected D days, but the difference between the two 108-day intervals comes out less than 0·3 day. The days of character ≮ 1·5, it should be remembered, are comparatively few, and accident plays a more than usually large part in the results derived from them.

§ 10. If the secondary and primary pulses had the same form, the comparison of the differences from the normal in the character figures for 27, 54,... days with the corresponding differences for day 0 would give a fair measure of the relative amplitudes. But such not being the case, comparisons were made which took account of 3 or 5 consecutive days, normally the central days, of each pulse. Table IX gives the 3-day results based on the difference pulses. Take, for example, the primary and the first secondary subsequent pulse for the 19 years. For the primary pulse the character differences (unit 0·001 character figure) for days − 1, 0 and + 1 were, respectively, 606 (302 + 304), 1222 (696 + 526) and 588 (365 + 223). For the secondary pulse the corresponding figures for days 26, 27 and 28 were 241, 334 and 236. The sum for days 26 to 28 thus bears to the sum for days − 1, 0 and + 1 the ratio 811 : 2416, or 0·336 : 1.

In some cases the 3 days centering at 81 or at 108 days give a smaller sum than 3 other consecutive days. For example, in the case of the third subsequent pulse for the 19 years, day 79 had a larger character difference than day 82. Taking the sum from days 80, 81 and 82, we get 0·078 for the ratio, while taking the sum from days 79, 80, 81, we get the larger value 0·085, which is given enclosed in brackets.

In the case of the first secondary pulse, the subsequent pulses in each case give a somewhat higher value for the ratio than the previous pulses, suggesting, as already explained, a slight tendency in the disturbance to increase on recurrence. But this phenomenon is not observed in the other secondary pulses, and may be accidental.

Taking the means from the subsequent and previous pulses, we see that the sunspot minimum group of years comes easily first, so far as the three first secondary pulses are concerned. Also low spot latitude conditions are decidedly more favourable than high spot latitude conditions.

There is by no means a regular reduction in the amplitude from pulse to pulse. The second secondary pulse is much larger compared to the first secondary than we should expect from the size of the first secondary as compared with that of the primary pulse. The fourth secondary pulse is also unexpectedly large as compared with the third.

TABLE IX.—3-Day Difference Pulses. Ratios of Secondary to Primary.

Group of years.	Subsequent pulses.				Previous pulses.				Mean of ratios, subsequent and previous.			
	First.	Second.	Third.	Fourth.	First.	Second.	Third.	Fourth.	First.	Second.	Third.	Fourth.
19 years	0·336	0·218	0·078 (0·085)	0·070	0·310	0·218	0·088 (0·090)	0·077	0·323	0·218	0·083 (0·087)	0·073
Years of low spot latitude	0·383	0·241	0·111	0·095	0·348	0·228	0·103	0·083	0·366	0·235	0·107	0·089
Years of high spot latitude	0·296	0·198	0·050 (0·075)	0·048 (0·050)	0·279	0·209	0·075 (0·083)	0·072	0·288	0·204	0·063 (0·079)	0·060 (0·061)
Years of few sunspots	0·409	0·281	0·134	0·083	0·379	0·241	0·135	0·069 (0·070)	0·394	0·261	0·135	0·076
Years of many sunspots	0·276	0·164	0·033 (0·067)	0·059 (0·075)	0·255	0·198	0·049 (0·065)	0·083	0·266	0·181	0·041 (0·066)	0·071 (0·079)

TABLE X.—5-Day Difference Pulses. Ratios of Secondary to Primary.

Group of years.	Subsequent pulses.				Previous pulses.				Mean of ratios, subsequent and previous.			
	First.	Second.	Third.	Fourth.	First.	Second.	Third.	Fourth.	First.	Second.	Third.	Fourth.
19 years	0·387	0·255	0·095 (0·099)	0·082	0·369	0·258	0·099 (0·106)	0·087	0·378	0·257	0·097 (0·102)	0·085
Years of low spot latitude	0·438	0·281	0·150 (0·152)	0·100	0·404	0·278	0·123	0·088	0·421	0·280	0·137 (0·138)	0·094
Years of high spot latitude	0·347	0·233	0·049 (0·087)	0·066 (0·067)	0·341	0·241	0·079 (0·099)	0·085	0·344	0·237	0·064 (0·093)	0·076
Years of few sunspots	0·465	0·304	0·141	0·075	0·444	0·265	0·126 (0·138)	0·062 (0·073)	0·455	0·285	0·134 (0·140)	0·069 (0·074)
Years of many sunspots	0·324	0·212	0·056 (0·081)	0·088 (0·093)	0·307	0·251	0·076 (0·081)	0·106 (0·114)	0·316	0·232	0·066 (0·081)	0·097 (0·103)

Table X differs from Table IX only in using 5 days in place of 3, the days centering at 0, 27, ... days. The values in brackets are derived from groups of 5 successive days which do not centre at 81 or at 108. It will be seen that the ratios in Table X are all sensibly larger than the corresponding ratios in Table IX. This is in accordance with what has been said as to the spreading of the disturbance in the secondary pulses.

In the case of the first secondary pulse the ratios are still invariably larger for the subsequent than for the previous pulse, but the excesses are smaller than in Table IX. The pre-eminence of the sunspot minimum years in the case of the first three secondary pulses is less than it was in Table IX, and in the case of the fourth pulse, the years of many sunspots actually take the first place. It was suspected that this might be a complex effect influenced by the varying size of the primary pulse in different types of years. To elucidate this point Table XI gives results derived from the different classes of disturbed days already used. It gives ratios calculated from both 3-day and 5-day sums for the positive subsequent pulses. The 5-day pulses are all taken centering exactly at 27, 54, 81 or 108 days. The groups are in order of diminishing disturbance in the primary pulse.

So far as the first three secondary pulses are concerned, the general tendency is clearly for the ratio to increase as the disturbance in the primary pulse diminishes, this being particularly apparent as between the days with characters less than 1·5 and the more disturbed days. But when it comes to the fourth secondary pulse the tendency is the other way. In this case the contribution from the disturbed days of least amplitude to the 3-day pulse is very small, and the contribution to the 5-day pulse is actually negative. This explains the prominent position in the case of the fourth secondary pulse taken by the years of many sunspots, which contain the great majority of the days with characters $\nless 1{\cdot}5$.

§ 11. In view of the unexpected nature of some of the phenomena, particularly those occurring in the third secondary pulse, it seemed desirable to ascertain how the pulses are made up. An excess in a mean character figure may signify a special concentration of high character figures, a general tendency to high rather than to low characters, or a special lack of low character figures. Table XII supplies information on this point. It shows exactly how character figures were assigned to the 7,305 days of the 20 years 1906 to 1925. It also shows the distribution of character figures on day $n + 27$, where n is representative of the 1,200 selected D or Q days of the 20 years. Days with characters from 1·1 to 1·5—*i.e.*, the lower grade of decidedly disturbed days—form one group. It is only slightly larger than the group formed by days of character 0·1. The first line of Table XII gives the actual number of days in each group for the 20 years. The differences in number between the groups, omitting the last two and the second, are not very big.

The second line gives to the nearest unit the results obtained by multiplying the numbers in the first line by 1,200/7,305; it thus shows the distribution to be expected in 1,200 days chosen by chance.

TABLE XI.—Positive Subsequent Pulses. Ratio to Primary. 19 Years 1906 to 1924.

Days forming primary pulse.	3-day pulse. First.	3-day pulse. Second.	3-day pulse. Third.	3-day pulse. Fourth.	5-day pulse. First.	5-day pulse. Second.	5-day pulse. Third.	5-day pulse. Fourth.
Character not less than 1·5	0·246 (0·279)	0·175 (0·197)	0·066 (0·077)	0·070 (0·075)	0·275	0·211	0·063	0·078
3 largest of 5 D days	0·316 (0·317)	0·210	0·084	0·075	0·345	0·250	0·079	0·091
All 5 D days	0·328	0·218	0·094	0·055	0·371	0·250	0·106	0·057
D days of character less than 1·5	0·388	0·250	0·111 (0·125)	0·042	0·445	0·278	0·136	0·037
2 least of 5 D days	0·355	0·233	0·113 (0·134)	0·016	0·427	0·249	0·145	—0·010

TABLE XII.—Occurrences of International Character Figures on All Days and on Days 27 Days Subsequent to the Selected D and Q Days.

International character.	0·0	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1·0	1·1 to 1·5	>1·5	Number of days.
All days, 20 years	472	933	644	581	490	449	444	476	519	569	426	1025	277	7,305
Expectation, 1,200 average days	78	153	106	95	81	74	73	78	85	93	70	168	46	1,200
Day 27 after selected D day	42	98	81	72	66	61	49	70	82	100	92	286	101	1,200
Day 27 after selected Q day	127	224	140	107	91	79	83	72	56	63	41	85	32	1,200
Ratios.														
Associated D day/expectation	0·54	0·64	0·77	0·75	0·82	0·83	0·67	0·90	0·96	1·07	1·31	1·70	2·22	
Associated Q day/expectation	1·64	1·46	1·32	1·12	1·13	1·07	1·14	0·92	0·66	0·67	0·59	0·50	0·70	
Associated D day/associated Q day	0·33	0·44	0·58	0·67	0·73	0·77	0·59	0·97	1·46	1·59	2·24	3·37	3·16	
Years of few sunspots.														
Expectation	94	173	117	96	81	71	74	73	89	93	67	139	33	1,200
Day 27 after selected D day	47	80	93	71	71	56	56	71	82	104	98	282	89	1,200
Day 27 after selected Q day	133	280	149	95	89	76	84	67	62	47	38	62	18	1,200
Years of many sunspots.														
Expectation	64	137	96	95	80	76	72	83	82	94	73	192	56	1,200
Day 27 after selected D day	38	113	71	73	62	65	44	69	82	96	87	289	111	1,200
Day 27 after selected Q day	122	178	133	116	93	82	82	76	51	76	44	103	44	1,200

The third and fourth lines give the distributions actually occurring on the central day ($n + 27$) of the first secondary positive and negative pulses. Results analogous to these appear in lines 8 to 10 for the group of years of few sunspots, and in lines 11 to 13 for the group of years of many sunspots. In each case a factor was applied to bring the total from the group up to 1,200. Lines 5, 6 and 7 give ratios calculated from the 20-year results in lines 2, 3 and 4. By " Associated D " is meant day 27 after a selected D day, and similarly for " Associated Q."

Taking the 20 years, we see that the 27-day positive pulse is mainly due to an excess in the number of days with characters 0·9 and upwards, combined with a deficiency in the number of days with low characters, especially 0·1 and 0·0. Similarly, the 27-day negative pulse is mainly due to a great excess in the number of days of low character figures, especially 0·0 and 0·1, combined with a large deficiency of days of characters 0·8 and upwards. The pulses are mainly due to the distribution of the days with characters less than 0·3 or greater than 0·9, *i.e.*, to the distribution of days with the same range of characters as we meet with in either the selected Q or the selected D days. Days of characters 0·7 and 0·8 are actually in deficiency in both groups of associated days.

The differences between 1,200 average days from years of few and years of many sunspots are less than would have been anticipated by any one familiar with the appearance of magnetic curves but unfamiliar with the De Bilt lists. Characters less than 0·3 are, however, decidedly more numerous in years of few sunspots, and characters greater than 1·0 are correspondingly more numerous in years of many sunspots. While 1,200 average days contain a much larger number of occurrences of characters 1·1 to 1·5 in years of many than in years of few sunspots, the difference between the number of these occurrences in 1,200 days following 27 days after selected D days in the two types of years is quite small.

On the other hand, the 1,200 days which follow 27 days after selected Q days show a very much larger number of occurrences of character 0·1 in years of few than in years of many sunspots. This is quite in accordance with the great prominence we have found in the 27-day pulses from the years of few sunspots.

A somewhat curious result brought out by the ratios in Table XII is that the number of disturbed days of character $> 1·5$, which follow selected Q days after 27 days, is considerably larger than we should have expected from the corresponding numbers for the days of lesser disturbances.

§ 12. The results reached in Table XII suggest that a good deal may be learned in a comparatively easy way by seeing how often selected D or Q days are followed after a given interval by other selected D or Q days. This is the object of Table XIII. It took all the selected D or Q days from February, 1906, to December, 1925—1,195 in each case—and ascertained how many of them followed 26, 27 or 28 days after a selected D or Q day.

If we had to do with pure chance, 1,195 average days of the 20 years would have, amongst the 1,195 days that preceded or followed them by any specified interval of days equal numbers, 196, of D and Q days. Thus, if there were no recurrence tendency, every entry in the first line of Table XIII for single days would be 196. As it is, when we take 26, 27, or 28 days for the interval following D days, we get a large excess of D days and a large deficiency of Q days ; while following Q days we get a large excess of Q days and a large deficiency of D days.

In the case alike of the 20 years and of the four groups of years, the number of sequences of D after D (D.D.) and of Q after Q (Q.Q.) is invariably greater for the 27-day than for the 26-day or 28-day intervals. Also, in general, there is remarkably little difference as between the 26-day and 28-day sequences. The Q after D (D.Q.) sequences are, with two exceptions, least for the 27-day interval, and in two cases out of five the D after Q (Q.D.) sequences are also fewest for that interval. The larger the excess of D.D. over D.Q. occurrences, the larger would the secondary positive pulse naturally be, and similarly with the excess of Q.Q. over Q.D. occurrences and the secondary negative pulse. Thus Table XIII suggests 27 days for the position of the crest of the first secondary pulses positive and negative for all the types of years considered.

§ 13. Table XIV is arrived at in a similar way to Table XIII, but it covers the first four secondary pulses, the primary days coming from the 19 years 1906 to 1924. To throw further light on the third secondary pulse, results were obtained separately for the D days, numbering 339, which had characters not less than 1·5. In their case the figures in brackets are from the days 27 days *earlier* than the selected D days.

It will be seen that within their respective groups, days 27, 54 and 108 are pre-eminent, whether we take the number of D.D. occurrences, the excess of D.D. over D.Q. occurrences, or the excess of Q.Q. over Q.D. occurrences. But day 80 is superior to day 81 as regards the number of D.D. occurrences or the excess of Q.Q. over Q.D. occurrences. As regards the latter item, its superiority to day 81 is enormous, and it is due in large measure to the very large number of Q.D. occurrences on day 81.

To make the results from the group of days of highest disturbance comparable with the others, multiplication by 3·36 (*i.e.*, 1140/339) is necessary. The outstanding phenomenon is the large amplitude of the fourth secondary pulse. Another interesting feature is the great excess in the number of D.Q. occurrences on day 81 over that on day 108. The number of occasions when Q days follow 81 days after D days of the highest class is quite outstanding.

Another interesting fact is the considerable difference between the results from day − 27 and day + 27. A tendency in the most disturbed D days to be the second rather than the first member of a sequence is consistent with the relatively large size of the first subsequent secondary pulse mentioned in § 5.

§ 14. Table XV aims, like Table XIII, at comparing results from different groups of years, but it is confined to sums of occurrences for 3 successive days centering at 27, 54, 81 and 108 days subsequent to the selected D and Q days. Also, as in Table XIV, the primary D and Q days come from the 19 years 1906 to 1924.

Table XIII.—Frequency of Occurrence of Selected D and Q Days 26, 27 or 28 Days subsequent to Selected D or Q Days.

Group of years.	D after D.				Q after D.				D after Q.				Q after Q.			
	26	27	28	3 days.	26	27	28	3 days.	26	27	28	3 days.	26	27	28	3 days.
20 years	298	350	293	941	155	122	130	407	103	106	144	353	271	315	276	862
Years of low latitude .	140	172	140	452	68	44	53	165	49	56	72	177	122	146	127	395
Years of high latitude .	158	178	153	489	87	78	77	242	54	50	72	176	149	169	149	467
Years of few spots . .	156	176	145	477	68	43	54	165	38	47	61	146	122	137	124	383
Years of many spots .	142	174	148	464	87	79	76	242	65	59	83	207	149	178	152	479

Table XIV.—Number of D and Q Occurrences, 1906 to 1924, on specified Days subsequent (or previous) to selected D or Q Days.

Primary days.	Nature of sequence.	26.	27.	28.	53.	54.	55.	80.	81.	82.	107.	108.	109.
19 years	D.D.	289	339	283	239	278	253	231	228	200	201	205	202
1,140 primary D days .	D.Q.	150	115	125	174	144	131	169	160	159	169	164	165
	D.D. less D.Q.	139	224	158	65	134	122	62	68	41	32	41	37
19 years	Q.Q. . . .	259	300	260	226	242	258	216	209	198	199	201	201
1,140 Primary Q days .	Q.D.	98	101	138	126	136	182	152	208	204	169	167	174
.	Q.Q. less Q.D.	161	199	122	100	106	76	64	1	— 6	30	34	27
339 D days of character	D.D. . . .		104 (116)			88			73			74	
≮ 1·5	D.Q.		41 (34)			47			55			44	
	D.D. less D.Q.		63 (82)			41			18			30	

To see how the results are obtained, consider the entries 47·9 and 20·5 for days 26 to 28 in the first line under the headings D after D and Q after D. The 19 years supplied 1,140 D days. The 3,420 (1,140 × 3) days which followed these after intervals of 26, 27 or 28 days included 911 D and 390 Q days; while the 6,940 days of the 19 years included 1,140 D and 1,140 Q days. Thus, in 3,420 average days we should expect to find 3,420×(1140/6940), *i.e.*, 561·8 D or Q days. This gives 29·6 of either class per annum. What we actually got per annum was 911/19, *i.e.*, 47·9 D days, and 390/19, *i.e.*, 20·5 Q days. The shorter groups of years were treated in an analogous way. Thus each entry in Table XV may be regarded as giving actually observed D and Q occurrences as compared with a pure chance occurrence of 29·6.

In the case of D.D. occurrences the years of low spot latitude always do better than the years of high spot latitude, and the years of few spots do better than the years of many spots. Except in the case of the fourth secondary pulse the years of few spots come first, and the years of many spots last. But in the case of the fourth secondary pulse the years of low spot latitude come first, and the years of high spot latitude fall short of the expectation value 29·6. The Q.Q. results seem little dependent on the type of year; the occurrences from days 107 to 109 for both the low latitude and the few spot years only slightly exceed expectation. The D.Q. occurrences all fall short of expectation. Their increase as we pass from one to the next subsequent pulse is much less marked than the corresponding decline in the D.D. occurrences. The largest D.Q. entry appears in the third pulse in years of many spots. In the case of the Q.D. occurrences the curious phenomenon that the third pulse presents invariably a larger number than the fourth should be noted. In two cases, years of high spot latitude and years of many sunspots, the expectation value is exceeded.

The size of the positive pulses will naturally depend on the excess of the D.D. over the D.Q. occurrences, and the size of the negative pulses on the excess of the Q.Q. over the Q.D. occurrences. These two sets of differences are given in the first 8 columns of Table XVI. The last four columns give the sum of the two differences, *i.e.* (D.D. — D.Q.) + (Q.Q. — Q.D.), which correspond to difference pulses.

Table XVI will be found to present a number of parallel features to Tables III and IV. In particular, the years of many sunspots provide in Tables III and XVI much the poorest values for the third positive pulse, while in the case of the third negative pulse Tables IV and XVI agree in putting the high spot latitude years last.

If the results in the last four columns of Table XVI are compared with the corresponding results based on 3-day sums of character figures in Table IX, it will be found that the order in which the groups of years present themselves in the two cases is absolutely identical so far as the first three secondary pulses are concerned. As regards the fourth secondary pulse, the two tables agree in putting the low spot latitude years first and the high spot latitude years last.

§ 15. Whilst decided differences have appeared between the contrasted groups of years, the accordance between years in the same group is not close. Thus, presumably,

TABLE XV.—Occurrences of D and Q Days, 3-Day Groups, 1906 to 1924.

—	D after D.				Q after D.				D after Q.				Q after Q.			
	26 to 28.	53 to 55.	80 to 82.	107 to 109.	26 to 28.	53 to 55.	80 to 82.	107 to 109.	26 to 28.	53 to 55.	80 to 82.	107 to 109.	26 to 28.	53 to 55.	80 to 82.	107 to 109.
19 years	47·9	40·5	34·7	32·0	20·5	23·6	25·7	26·2	17·7	23·4	29·7	26·8	43·1	38·2	32·8	31·6
Years of low latitude . .	49·8	42·0	37·6	35·1	18·6	21·3	25·4	27·1	19·9	24·4	29·1	25·7	43·8	37·9	32·7	30·3
Years of high latitude .	46·3	39·2	32·1	29·2	22·3	25·7	25·9	25·4	15·8	22·4	30·2	27·9	42·5	38·5	32·9	32·8
Years of few sunspots .	53·4	45·8	39·8	33·2	18·0	23·3	22·9	27·1	16·0	22·2	28·6	26·9	42·7	37·6	31·8	30·2
Years of many sunspots	43·0	35·8	30·1	30·9	22·8	23·9	28·2	25·4	19·3	24·4	30·7	26·8	43·5	38·8	33·7	32·9

TABLE XVI.—Occurrences of D and Q Days, 3-Day Groups, 1906 to 1924.

—	D.D. less D.Q.				Q.Q. less Q.D.				D.D. + Q.Q — D.Q. — Q.D.			
	26 to 28.	53 to 55.	80 to 82.	107 to 109.	26 to 28.	53 to 55.	80 to 82.	107 to 109.	26 to 28.	53 to 55.	80 to 82.	107 to 109.
19 years	27·4	16·9	9·0	5·8	25·4	14·8	3·1	4·8	52·8	31·7	12·1	10·6
Years of low latitude . . .	31·2	20·7	12·2	8·0	23·9	13·5	3·6	4·6	55·1	34·2	15·8	12·6
Years of high latitude . .	24·0	13·5	6·2	3·8	26·7	16·1	2·7	4·9	50·7	29·6	8·9	8·7
Years of few sunspots . .	35·4	22·5	16·9	6·1	26·7	15·4	3·2	3·3	62·1	37·9	20·1	9·4
Years of many sunspots . .	20·2	11·9	1·9	5·5	24·2	14·4	3·0	6·1	44·4	26·3	4·9	11·6

other things than the number or latitude of sunspots are concerned. With a view to the discovery of what the other influences may be, it seems desirable to give for each year data sufficient to show its peculiarities. This is the aim of Tables XVII, XVIII and XIX.

TABLE XVII.—Subsequent and previous (reversed) Difference Pulses (unit 0·001 character figure).

Year.	Primary pulse.				1st secondary pulse.					2nd secondary pulse.				
	0.	1.	2.	3.	25.	26.	27.	28.	29.	52.	53.	54.	55.	56.
1907	2360	1173	122	—147	117	377	372	150	118	57	348	528	462	297
1908	2373	1302	210	—275	458	835	765	495	193	503	393	340	377	280
1909	2420	1220	333	—143	—123	138	458	513	395	233	125	267	283	280
1910	2330	1178	282	— 75	43	457	713	532	257	— 60	148	230	270	203
1911	2397	1137	148	—207	342	782	1065	815	250	100	597	925	678	188
1912	2117	958	—80	—308	177	498	553	237	— 30	157	530	523	125	— 42
1913	2193	1140	417	— 20	413	768	927	663	423	13	230	390	367	240
1914	2237	973	220	—215	188	173	82	192	317	—183	—10	13	162	72
1915	2617	1272	47	— 43	398	555	853	720	440	137	232	512	600	365
1916	2313	1288	225	—163	—175	13	340	357	147	65	278	285	208	253
1917	2647	1175	223	—172	200	353	327	268	195	440	410	67	5	—145
1918	2763	1525	415	— 75	397	520	513	335	168	272	438	673	475	47
1919	2757	1233	—25	—425	88	332	575	418	178	—183	—30	332	143	—118
1920	2497	1083	57	—368	67	407	830	327	77	— 60	123	277	103	35
1921	2373	1037	190	112	263	180	303	258	37	258	203	318	310	177
1922	2520	1272	393	— 55	407	1037	1197	840	577	455	693	793	902	375
1923	2297	1302	282	—225	247	640	967	567	—112	295	667	808	412	—145
1924	2600	1188	210	—112	127	577	732	585	432	—113	143	185	223	138

Year.	3rd secondary pulse.					4th secondary pulse.				
	79.	80.	81.	82.	83.	106.	107.	108.	109.	110.
1907	63	68	87	62	13	7	— 20	87	153	220
1908	505	573	422	178	212	273	427	560	397	307
1909	133	172	— 8	— 43	—143	207	260	102	— 58	—202
1910	—338	—260	— 3	257	135	—265	—245	— 37	45	— 85
1911	—115	183	532	600	275	—193	13	212	260	45
1912	222	280	153	— 3	— 80	187	163	20	— 95	—147
1913	117	285	313	128	—192	— 70	160	185	83	—223
1914	— 63	175	—215	—128	—113	—230	—362	—273	—150	— 70
1915	38	87	140	230	145	— 33	77	213	182	45
1916	— 10	138	143	75	78	32	—148	—107	— 30	92
1917	92	5	—183	—392	—190	—255	— 77	120	190	242
1918	— 12	— 42	— 83	—107	—102	243	318	333	193	20
1919	25	242	212	— 48	—105	—140	— 2	— 2	37	— 77
1920	—145	— 87	—288	—127	5	—122	—133	—112	5	172
1921	173	— 45	— 92	—158	—165	— 5	133	5	225	20
1922	412	587	743	593	390	238	567	610	445	272
1923	353	447	462	45	—468	432	547	313	—215	—417
1924	102	270	305	133	—138	258	232	180	125	227

Table XVII gives for years 1907 to 1924, for the primary and four secondary pulses, results obtained by combining the subsequent difference pulses with the previous difference pulses reversed, *i.e.*, results of the same kind as appear in Table VII. As the primary pulses are absolutely symmetrical, the results for days − 3, − 2 and − 1 are omitted.

In 1907 the first and third secondary pulses suggest an interval shorter than 27 days, but the second pulse goes slightly and the fourth pulse markedly in the opposite direction. In 1908 the first three pulses suggest an interval well under 27 days, but the fourth pulse, which is specially well developed, suggests almost exactly 27 days. In 1909 the first and second pulses suggest more than 27 days, but the remaining pulses less. In 1910 the second pulse, and to a minor extent also the first, suggest over 27 days; the other pulses are very irregular. In 1911 the third and fourth, and to a minor extent the second pulse, suggest an interval in excess of 27 days. In 1912 all the pulses are favourable to a shorter value of the interval. In 1913 the maxima fall on days 27, 54, 81 and 108. In 1914, even in the 27-day pulse, the maxima are altogether too uncertain. In 1915 all the pulses suggest an interval longer than 27 days; still, days 27 and 108 come clearly first in their respective groups. In 1916 the first pulse suggests a longer value for the interval, but the second and third pulses do not; the fourth pulse seems to be negative. In 1917 we have maxima on days 26 and 52, *i.e.*, decidedly early, and a maximum, not shown in the table, appears on day 78; but of the days from 104 to 112, day 110 supplies the largest positive value, while days 105 and 107 provide negative values. In 1918 a maximum appears on day 26, but the subsequent maxima which are recognisable appear on days 54 and 108. In 1919 a maximum appears on day 80, but days 53 and 106 to 108 supply negative values. In 1920 and 1921 maxima appear on days 27 and 54, and the third and fourth pulses are altogether irregular. In 1922 the maximum on day 55 favours a long value for the interval, but the other pulses do not support this. In 1923, while maxima fall on days 27, 54 and 81, all the pulses, especially the fourth, suggest an interval less than 27 days. In 1924 the second pulse points one way, the third and fourth another. 1906 and 1925 were omitted from the table because we had only a subsequent pulse for the one year and a previous pulse for the other. In 1906 the first three subsequent pulses suggest a short value for the interval, maxima appearing on days 53 and 79. But in the fourth secondary pulse, which is fairly developed, a maximum appears on day 110. In 1925 the difference pulses have crests on days − 28, − 56 and − 83, suggestive of a long value of the interval, but the fourth pulse is too irregular for any inference to be drawn. On the whole, 1906, 1908, 1912, 1917 and 1923 suggest shorter, and 1911, 1915 and 1925 longer, values for the interval; but in no case is the evidence very strong for an interval departing decisively from 27 days.

§ 16. Inferences as to the relative amplitudes of the pulses in different years could be drawn from Table XVII, but this can be more readily done from Table XVIII, which contains 3-day sums from difference pulses. The subsequent and previous pulses

TABLE XVIII.—3-Day Sums (unit 0·001 character figure).

Year.	Primary pulse.	Difference pulses, subsequent.				Difference pulses, previous.				Subsequent and previous pulses combined.			
	−1 to +1 day.	26 to 28 days.	53 to 55 days.	80 to 82 days.	107 to 109 days.	−26 to −28 days.	−53 to −55 days.	−80 to −82 days.	−107 to −109 days.	26 to 28 days.	53 to 55 days.	80 to 82 days.	107 to 109 days.
1906	2378	695	523	215	300	—	—	—	—	—	—	—	—
1907	2353	515	610	155	138	383	728	62	82	898	1338	217	220
1908	2488	1095	663	667	763	1000	447	507	620	2095	1110	1174	1383
1909	2430	478	275	−23	195	632	400	143	108	1110	675	120	303
1910	2343	1010	387	48	−248	692	262	−55	12	1702	649	−7	−236
1911	2335	1417	1208	818	390	1245	992	497	95	2662	2200	1315	485
1912	2017	632	658	197	218	657	520	233	−130	1288	1178	430	88
1913	2237	1083	412	247	157	1275	575	480	272	2358	987	727	429
1914	2092	287	187	−147	−468	160	−22	−22	−317	447	165	−169	−785
1915	2580	1163	627	167	152	965	717	290	320	2128	1344	457	472
1916	2595	333	152	172	−153	377	620	185	−132	710	772	357	−285
1917	2498	460	122	−292	95	488	360	−278	138	948	482	−570	233
1918	2907	713	833	5	330	655	753	−237	515	1368	1586	−232	845
1919	2612	572	−18	33	−213	753	463	372	247	1325	445	405	34
1920	2332	922	360	−285	−117	642	143	−217	−123	1564	503	−502	−240
1921	2223	568	352	−92	215	173	480	−203	148	741	832	−295	363
1922	2532	1465	1242	1005	828	1608	1147	918	793	3073	2389	1923	1621
1923	2450	1108	943	330	155	1065	943	623	490	2173	1886	953	645
1924	2488	918	448	375	473	975	103	333	63	1893	551	708	536
1925	2477	—	—	—	—	543	373	418	322	—	—	—	—

are given separately, as it is of interest to see how far they support one another, but the two sets of results are also combined for the years for which both were available.

There is substantial variation in the amplitudes of the primary pulses, the maximum in 1918 (the year after sunspot maximum) being 44 per cent. larger than the minimum in 1912 (the year before sunspot minimum), but the variations in the amplitudes of the secondary pulses are very much larger. 1922, a year of low spot latitude immediately preceding sunspot minimum, comes only fifth as regards the amplitude of the primary pulse, but it supplies the largest value in every one of the subsequent columns. In the case of the combined pulses, the second, third and fourth pulses in 1922 exceed the first, second and third pulses, respectively, in every other year, with the exception of 1911. Next to 1922, so far as the first three secondary pulses are concerned, comes 1911, also a year of low spot latitude and few sunspots, preceding sunspot minimum by two years.

The fourth secondary pulses in 1911, especially the previous pulse, are, however, less well developed. In 1908, on the other hand, the fourth secondary pulse is exceptionally well developed. It is second only to that of 1922, and in its own year is superior to the second and third pulses. 1923, a year of sunspot minimum, surpasses 1908 so far as the first two secondary pulses are concerned; and its third and fourth pulses, though inferior to those of 1908, are very fairly developed. 1918 seems rather a curious year. Its second and fourth secondary pulses are amongst the largest, but the third subsequent pulse is practically non-existent, and the third previous pulse supplies a substantial negative value. 1914, a year immediately following sunspot minimum, is the exact opposite of 1922. Its third and fourth secondary pulses, both subsequent and previous, and its second previous pulse come negative, and even its first secondary pulse is very poorly developed. 1910 and 1920 start well, so far as their first secondary pulses are concerned, but their second pulses are poor, and their third and fourth pulses negative. 1907, the second year of its cycle as regards sunspot number, has its second secondary pulse well developed, but the others are rather poor. 1917, a prominent year of sunspot maximum, has all its secondary pulses poorly developed, the third supplying the largest negative value in its class. It will be seen that, in all, six years supplied negative values for the combined third secondary pulses, as against four for the combined fourth secondary pulses. Thus the occurrence of a negative value does not necessarily mean that the 27-day interval cannot be traced farther. We should certainly infer from the general appearance of Table XVIII that it could be traced for several pulses beyond the limits of the table, especially in the years 1908 and 1922.

§ 17. Table XIX attacks the same problem as Table XVIII, but from a different angle. It relates to subsequent pulses only, but treats the positive and negative pulses separately, as well as in combination. Take, for example, the first positive and negative pulses of 1906. The 60 selected D days have in all 180 days following them at intervals of 26, 27 or 28 days. The number of selected D days included in these 180 days exceeded by 15 the number of selected Q days. This figure heads the first column. The 60 selected Q days had amongst the 180 days which followed them at intervals of 26, 27 or 28 days, 48 more Q days than D days, and 48 heads the fifth column. The sum of these two excesses, 63, heads the 9th column. These figures, 15, 48 and 63, will naturally be larger according as the corresponding positive, negative and difference pulses, respectively, are the better developed. We see that precisely, as in Table XVIII, 1922 is pre-eminent in all its difference pulses. It owes its premier position to the fact that its positive and negative pulses are both well developed.

TABLE XIX.—D and Q Occurrences in 3-Day Groups.

Year.	Positive pulses. Excess of D.D. over D.Q. occurrences.				Negative pulses. Excess of Q.Q. over Q.D. occurrences.				Difference pulses. (D.D.—D.Q.) + (Q.Q.—Q.D.).			
	26 to 28 days.	53 to 55 days.	80 to 82 days.	107 to 109 days.	26 to 28 days.	53 to 55 days.	80 to 82 days.	107 to 109 days.	26 to 28 days.	53 to 55 days.	80 to 82 days.	106 to 109 days.
1906	15	20	5	− 3	48	9	23	13	63	29	28	10
1907	13	21	0	1	19	32	9	12	32	53	9	13
1908	37	13	23	24	44	27	23	28	81	40	46	52
1909	12	0	−11	− 4	16	19	− 3	− 3	28	19	−14	− 7
1910	21	8	− 1	− 9	42	8	6	− 2	63	16	5	−11
1911	52	50	38	12	42	26	7	− 5	94	76	45	7
1912	30	25	18	11	1	7	1	− 3	31	32	19	8
1913	60	18	24	5	29	12	− 9	2	89	30	15	7
1914	1	− 6	− 2	−17	13	10	0	−13	14	4	− 2	−30
1915	19	28	− 1	10	36	19	12	23	55	47	11	33
1916	17	2	17	− 1	13	20	5	−11	30	22	22	−12
1917	12	−11	− 9	19	25	3	−10	1	37	− 8	−19	20
1918	17	23	0	12	18	24	−15	3	35	47	−15	15
1919	29	21	17	− 1	15	−10	−11	− 3	44	11	6	− 4
1920	33	2	−22	− 2	8	1	− 3	− 2	41	3	−25	− 4
1921	15	23	5	12	− 1	8	−14	4	14	31	− 9	16
1922	58	44	42	29	51	35	26	28	109	79	68	57
1923	45	36	12	3	33	18	5	3	78	54	17	6
1924	30	4	16	9	27	14	7	16	57	18	23	25
Totals . .	516	321	171	110	479	282	59	91	995	603	230	201

Again, as in Table XVIII, 1911 comes next to 1922 as regards the first and second secondary pulses, but its fourth secondary pulse is comparatively poor, and this is true of both the positive and the negative pulse, especially the latter. 1908 is again inferior to 1911 and 1913, so far as the first secondary pulse is concerned, but its fourth difference pulse is second only to that of 1922, and exceeds its second and third pulses. It is strong both in its positive and its negative pulses. As in Table XVIII, 1914 makes the poorest show of all, and the positive pulses fail worst. Other poor years are 1909 (especially for the positive pulse), 1917, 1920 and 1921. 1913 starts off exceedingly well, especially for the positive pulse; but the third and fourth pulses, especially the negative ones, are poorly developed. In short, the agreement between Tables XVIII and XIX is exceedingly good. This is all the more satisfactory, because the computations involved in the construction of Table XIX are comparatively light. If the totals in the last four columns are multiplied, respectively, by 2·5, 4, 10 and 12, the results are roughly equal (*cf.*, figs. 3 and 4).

§ 18. In view of the contrast presented by 1920 and 1921 on the one hand and 1922 and 1923 on the other, these pairs of years suggested themselves for the investigation summarised in Tables XX and XXI. The days are arranged in "months" of 27 days,

TABLE XX.—Distribution of Quiet and Disturbed Days, 1920–1921.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	
I				1	0				11	11	13		0						2		12						0	I
II	10					0	1		(1)		10		1							16	14	9					1	II
III	16					0		1	0		19									18	(10)				(1)			III
IV	20	20	18					1	1	1	1				13						1		2	2	19		14	IV
V	12						11	(1)	(2)		0			12					1	0				0	(1)	16	11	V
VI	12							1	1					13					1		10				0		16	VI
VII	11			0				2				1						8	8			0	1		(1)		12	VII
VIII					11			14	10					1		11					0	0			(2)	0		VIII
IX			0		11	11			14					1				12	13					1	1			IX
X				16	(14)		2			14			1					(11)			2	2	17		(3)		1	X
XI		19	17	(11)	13		3							15				1						1	1			XI
XII	10	13		11	(9)		0						10	13		1		1				1			11			XII
XIII		(10)		1	1		16	12				0			17	14	11					1	1					XIII
XIV					(1)	0			11	17				1		9	2						2	8	10			XIV
XV	1	2			14					2		9								9	9			12	9		1	XV
XVI	1			1	(9)			1			9				1										1			XVI
XVII	1							13		2		1	1	14	13			13		13								XVII
XVIII	1	1		1					12	16					14	12	(10)						1	1		17		XVIII
XIX					0	1	0					(14)	19	20	20	20	(16)	(9)	(16)	18	(13)				1			XIX
XX			1					10	1	11		12	10	(9)					2			1	2				9	XX
XXI		1							1					10	10	12		1				9	10		3			XXI
XXII	1				1							0		12		11	11			2		(10)				(10)		XXII
XXIII							1		0	11			0	12			18						11			1	0	XXIII
XXIV	1										11	0	0	(0)		14	15				2				10	19		XXIV
XXV		14	11				1	1	1	1		11												0	1	0		XXV
XXVI	13										16	17	12					11				1		0		(2)		XXVI
XXVII				0	0					14	16			13			1	0	0							17	13	XXVII

each "month" occupying one row. The month is distinguished by the Roman numeral, the day by the Arabic. In Table XX, 1920 extends from I, 1 (January 1), to XIV, 15 (December 31). The last day in this table, XXVII, 27, represents December 29, 1921, which happened to be the latest D or Q day in the year. The entries in the table are primarily the international character figures, multiplied by 10, for all the selected D and Q days. Data for certain additional days are included, but for distinctive purposes are enclosed in brackets. The value of the figure suffices to distinguish D from Q days, no D day in the years included having a character figure less than 0·8; and no Q day a character greater than 0·3.

Before considering details, one peculiarity of D and Q days calls for notice. Each calendar month contains 5 D and 5 Q days. If a disturbed month follows a quiet month, several days in the second month may fail to be D days, though more disturbed than D days of the first month. When summing up sequences, for example in Table XIII, we have confined ourselves strictly to the selected D and Q days, but it appeared desirable to show in Tables XX and XXI days which, though not selected days, were of similar character, when there was ground for regarding these days as possible members of a sequence. No supernumerary disturbed day introduced has a character less than 0·9, while a good many real D days had this character, and one XIV, 24 (January 9, 1921), had character 0·8. The highest character, 0·3, possessed by a supernumerary quiet day, X, 25 (September 24, 1920), is paralleled within nine days by a real Q day,

Table XXI.—Distribution of Quiet and Disturbed Days, 1922–1923.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	
XXVIII					1	2				15	13						0						0	0		18	15	XXVIII
XXIX						13			14				2		12		2	12			14	15						XXIX
XXX		1	1	0				12				15		1	1	1	(12)			18	19							XXX
XXXI			1	1						14					0	0	0	1		13	(12)		13					XXXI
XXXII			1			16		13		14						0	0				13	13	13					XXXII
XXXIII		1	14					12								(0)	0	0			(11)		13					XXXIII
XXXIV	1	2				0	13	11							1	2				14	14	12						XXXIV
XXXV		1			1	0				12					1	1				15	14	15					3	XXXV
XXXVI	2								16	14	13	13			1	1					12					3		XXXVI
XXXVII				3					14	15	12	(11)				19					(2)	14		2	2	1		XXXVII
XXXVIII	1					2				19	13	12	11			(2)					1		2	1			0	XXXVIII
XXXIX									16	12	12	12			0	0			11			0					1	XXXIX
XL								0			13				1	(1)		9		0		10				11	9	XL
XLI	0	(0)					0	1			13					(2)		(2)					1	1				XLI
XLII		10					1	1	13	10						0		11	9									XLII
XLIII		1			1	1				11				1		1		18	16	17	10							XLIII
XLIV		1	1	1	1				10	(9)							0	19	18	14	11						1	XLIV
XLV		(2)	0	(1)						9	11				1			(9)	12	12	10			1	1			XLV
XLVI		1	(1)	10									1	1				16	13				1		1			XLVI
XLVII		(1)	12	11					10					1	1			17	12			1	1			10		XLVII
XLVIII		0		(2)				12					1		9			14	9			0	1			9		XLVIII
XLIX		0		11	0								1		11	9		8					(1)		9			XLIX
L		(1)	0			0			7				1	0	(1)								0	0	9	9		L
LI					0					0	0				17	20	12				0	0	(0)					LI
LII							19	20	16	14	10				0	0						0	(8)		13			LII
LIII	1					1		9			0					0		8			0	(1)	11		11			LIII
LIV	0					2										0				2	1	10	11	10	16	11		LIV

XI, 7 (October 3). It will be seen that several of the supernumerary quiet days had characters 0·1, one had even 0·0. The fact is that the size of the character figure was not in all cases the only criterion taken into account at De Bilt in the selection of Q days. For instance, XXIV, 1 (September 13, 1921), with character 0·1, is a Q day, while XXIV, 14 (September 26), with character 0·0, is not. But September 24 and 25 also had character 0·0, and to have had three successive days, coming near the end of the month, all Q days would have led to a rather unbalanced choice. For the purpose now in view, September 26 is equally significant with September 24 and 25. Take again X, 5 (September 4, 1920). September 4 has the same character, 1·4, as September 9. It may be regarded as pure accident that September 9 was selected a D day, in preference to September 4. As it happened, September 9 was an isolated disturbed day, while September 4 was one of a sequence, with a higher character than the adjacent members of the sequence. To have left out September 4, and so suggested a break in the sequence, seemed absurd.

Table XX shows a few fair sequences of D days, the longest being from II, 1 (January 28, 1920), to VII, 1 (June 11, 1920), but there is a marked tendency for D and Q days to occur as if members of the same sequence. Take, for example, day 11. We have three successive D days in "months" I, II and III, the last having the very high character 1·9, followed immediately by Q days with characters 0·1 and 0·0. One of the most interesting cases is day 25. Starting with a 0·1 in "month" III, we have the very high character 1·9 in "month" IV, then seven successive days of quiet day character, finishing up with XII, 25, a selected D day. Subsequent to this there are 7 other selected D or Q days out of a possible of 15 days, but the gaps left were filled by days of neither D nor Q character. Day 26, "months" XXII to XXVII is an example of violent alternations of character.

1921 included much the most disturbed period of recent times. Its centre is represented by the four days XIX, 13 to 16 (May 13 to 16), with characters of 1·9 or 2·0. The restriction of D days to 5 a month led to only the 20th appearing as a D day in addition to the 4 days already mentioned. But all the days from the 12th to the 21st, with the possible exception of the 18th, were well up to the ordinary D standard. On the usual theory, this would suggest active disturbance extending over quite a third of the sun's perimeter. The disturbance in its earlier stages was of quite a different order of intensity from any shown in adjacent months, but several of the days figure in Table XX as members of a sequence. Three of the days, after a lapse of 5 "months," have 0's in their train.

Table XXI follows exactly the same lines as Table XX, and to facilitate the tracing of sequences extending from 1921 to 1922, time is counted as in the previous table from January 1, 1920, as day I, 1. What first catches the eye in Table XXI is the run of D occurrences in days 9 to 12 and 18 to 21.

It is little wonder that, in 1922, the D pulses in Tables XVIII and XIX came out strong. But, even here, there are a number of rather marked transitions from disturbed to quiet, or quiet to disturbed, conditions. On day 16 we have a great preponderance of days of extreme quietness, but XXXVII, 16 (September 14, 1922), one of the most disturbed days of the year, comes in the middle of an otherwise unbroken sequence of quiet days. Again LI, 16 (September 27, 1923), rivalled as a disturbed day in 1923 only by October 16, is followed by a sequence of three days of character 0·0.

Other years were treated in the same way, but the results are not reproduced here, as they merely supported the inferences naturally drawn from what precedes. If magnetic disturbance on the earth is associated with limited disturbed areas on the sun, and if the rotation period of these areas is 27 days, as the numerous data of this paper suggest, then a solar area may continue highly disturbed for a number of months, or it may go through a succession of alternate states of high disturbance and unusual quietness. In some years a forecast based on a single D day is just about as likely to supply a specially quiet day as a really disturbed day. In other years, with reasonable luck, one might forecast a succession of several disturbed days.

§ 19. It remains to consider whether the international character figures support Dr. DESLANDRES' contention that magnetic disturbances have recurrence periods which are submultiples of 27 days. Mean values of the character figures were calculated for the 35 successive days $n-3$ to $n+31$, where n is the representative D or Q day of the 20 years 1906 to 1925. The results are given in Table XXII as differences from 0·619, the normal figure for the 20 years, 0·001 character being employed as the unit. The result under each day, for either the D or the Q primary and associated days, represents a mean from 1,200 days, *i.e.*, 84,000 data are employed in the table.

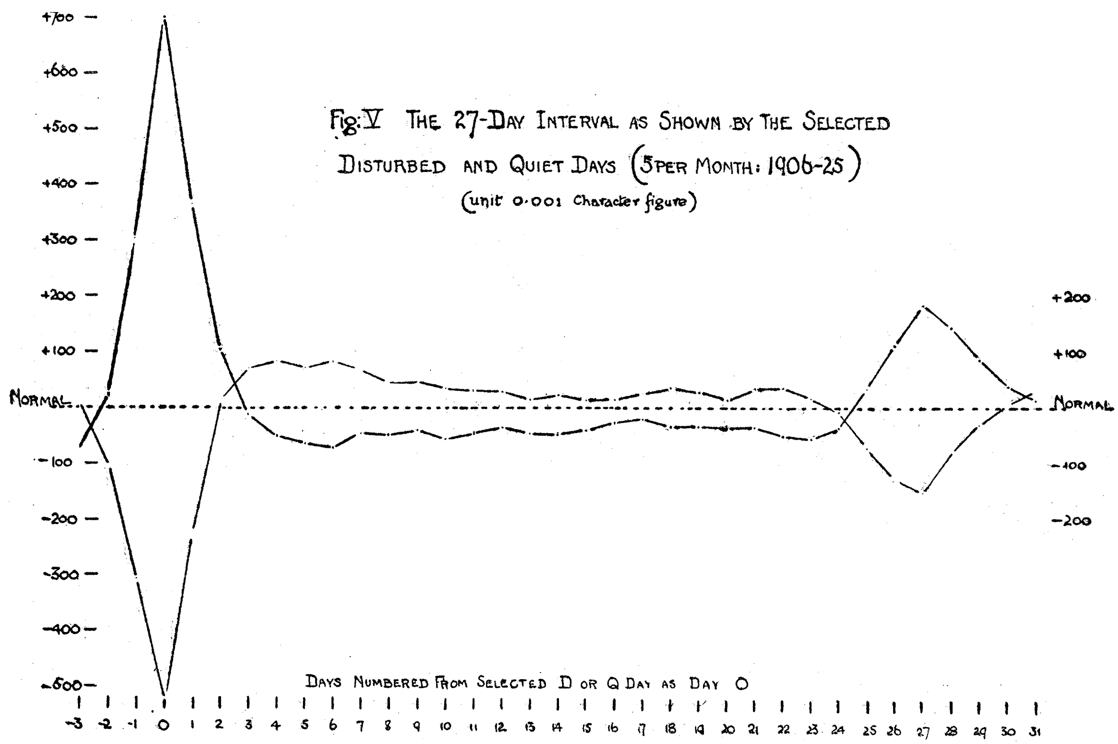

FIG. 5.

The general nature of the results is most easily followed by reference to fig. 5. In the case of the disturbed and associated days, after the conclusion of the primary pulse, there is a deficiency in every day from $n+3$ to $n+24$. Between these points there is only a trifling amount of undulation in the curve. After attaining a minimum on day $n+6$, the character figure rises on the whole up to day $n+17$, when it is short of the normal by only 0·20. Thereafter it falls on the whole to a second minimum on day $n+23$, not quite so low as that on day $n+6$. The rise to the secondary pulse

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TABLE XXII.—Character Figures on D and Q Days and their Associated Days 1906 to 1925. Differences from the Normal (unit 0·001 character figure).

Day.	$n-3$.	$n-2$.	$n-1$.	n.	$n+1$.	$n+2$.	$n+3$.	$n+4$.	$n+5$.	$n+6$.	$n+7$.	$n+8$.
D and associated days	−71	+ 11	+301	+700	+362	+103	−14	−49	−61	−69	−45	−47
Q and associated days	+ 5	−103	−304	−527	−224	+ 14	+67	+81	+72	+81	+68	+45

Day.	$n+9$.	$n+10$.	$n+11$.	$n+12$.	$n+13$.	$n+14$.	$n+15$.	$n+16$.	$n+17$.	$n+18$.	$n+19$.	$n+20$.
D and associated days	−40	−57	−44	−38	−45	−46	−40	−26	−20	−33	−33	−35
Q and associated days	+44	+38	+31	+29	+16	+21	+13	+16	+25	+39	+28	+17

Day.	$n+21$.	$n+22$.	$n+23$.	$n+24$.	$n+25$.	$n+26$.	$n+27$.	$n+28$.	$n+29$.	$n+30$.	$n+31$.
D and associated days	−35	−49	−54	−37	+17	+114	+182	+144	+86	+40	+15
Q and associated days	+36	+38	+19	− 6	−73	−126	−149	−90	−29	+ 4	+32

TABLE XXIII.—Distribution of Characters on Days $n+9$ and $n+18$, where n is Representative D Day (20 years).

Character.	0·0.	0·1.	0·2.	0·3.	0·4.	0·5.	0·6.	0·7.	0·8.	0·9.	1·0.	1·1 to 1·5.	>1·5.	Total days.
Days $n+9$	83	169	129	99	88	67	72	72	83	96	59	143	40	1,200
Days $n+18$	81	147	118	98	84	84	82	72	110	93	69	130	32	1,200
Average days	78	153	106	95	81	74	73	78	85	93	70	168	46	1,200

then begins, and the pulse is not quite terminated on day $n + 31$. Neither the numbers nor the figure suggest periods such as 4·5, 9, 13·5, 18 or 22·5 days.

In the case of the quiet and associated days, character has risen above the normal even as early as day $n + 2$, and it continues to rise to a maximum between days $n + 4$ and $n + 6$. Thereafter there is, on the whole, a gradual decline, until about day $n + 15$. The fall to the secondary negative pulse seems to set in rather earlier than the rise to the corresponding positive pulse, and the secondary negative pulse seems to end sooner than the secondary positive pulse. In short, the secondary positive and negative pulses differ somewhat in the same way as do the corresponding primary pulses.

§ 20. In view of the weight naturally attracting to Dr. DESLANDRES' opinions, it seemed well to investigate the question further. It was accordingly investigated what the distribution of character figures actually is on days $n + 9$ and $n + 18$, where n is representative of the 1,200 selected D days of the 20 years. It will be noticed that 9 and 18 are the only whole numbers arising from $i \times 27/6$, where i is integral. The results appear in Table XXIII, with, for comparison, the results already given in Table XII for 1,200 chance days of the 20 years.

If Dr. DESLANDRES' views were correct, so far as large disturbances at least are concerned, whatever else the figures in Table XXIII might show, they ought, presumably, to show an excess in the entries in the last three columns as compared with average days. The exact reverse is the case.

As a matter of fact, the days $n + 9$ included only 169 selected D days as against 227 selected Q days, a *deficiency* of 58, while the days $n + 18$ included only 143 D days, as against 209 Q days, a deficiency of 66.

§ 21. The absence of marked undulations in the curves of fig. 5, between days $n + 3$ and $n + 23$, could not have been foretold, but the substantial difference actually seen between the intermediate portions of the two curves in fig. 5 was a necessity of the case. Taking the D and associated days, let us suppose the representative day n to be at the centre of a 31-day month, and let us assume—what is not quite true—that symmetry of characterisation exists with respect to day n. Then we have 31 days, of which the central five give a total excess above the normal of 1,477 units.

It follows that, on the average, each of the remaining 26 days—which would extend on the positive side to day $n + 15$—must be about 57 units short of the normal. Days subsequent to $n + 15$ would, in the above hypothetical case, come from the next month. They, too, up to day $n + 24$, will have to provide a deficiency, helping to make up for the secondary positive pulse. Thus the facts that the portion of curve from day $n + 3$ to day $n + 23$, connecting the primary and secondary pulses, falls below the normal, and that the deficiency is greater for the portion of curve immediately following the primary pulse, are quite in accordance with anticipations. Similar reasoning leads us to anticipate that the portion of curve connecting the primary and secondary negative pulses will lie above the normal, throughout at least the greater part of its course.

But the aggregate deficiency in the primary negative pulse is substantially less than the corresponding excess in the primary positive pulse, being only 1,144, as against 1,477 units.

Thus, *a priori*, we should have expected that from day $n + 3$ to day $n + 23$, the curve from the associated Q days would have lain nearer the normal than the curve from the associated D days. This is true for days $n + 10$ to $n + 16$, but is somewhat conspicuously untrue for days $n + 3$ to $n + 7$.

TABLE XXIV.—Occurrences of D and Q Days from 4 to 6 Days after Q Days.

Year.	D after Q.			Q after Q.			3-day sums. Absolute numbers.		3-day sums. Ratio to expectation.	
	n+4.	n+5.	n+6.	n+4.	n+5.	n+6.	Q.D.	Q.Q.	Q.D.	Q.Q.
1906	15	9	8	2	4	7	32	13	1·08	0·44
1907	13	15	15	1	1	3	43	5	1·46	0·17
1908	16	13	6	7	9	10	35	26	1·19	0·88
1909	11	12	8	6	9	9	31	24	1·05	0·81
1910	13	15	18	6	4	7	46	17	1·55	0·58
1911	12	7	11	7	8	5	30	20	1·02	0·68
1912	9	8	8	8	8	4	25	20	0·84	0·68
1913	10	9	12	10	9	10	31	29	1·05	0·98
1914	6	9	9	7	8	8	24	23	0·81	0·78
1915	8	13	15	6	4	6	36	16	1·22	0·54
1916	9	9	12	8	7	5	30	20	1·02	0·68
1917	15	15	13	7	3	6	43	16	1·46	0·54
1918	14	14	15	4	3	5	43	12	1·46	0·41
1919	11	12	13	6	8	3	36	17	1·22	0·58
1920	14	12	11	6	3	10	37	19	1·25	0·64
1921	12	13	5	5	2	4	30	11	1·02	0·37
1922	13	22	20	5	5	4	55	14	1·86	0·47
1923	15	16	13	3	8	5	44	16	1·49	0·54
1924	10	13	15	5	5	8	38	18	1·28	0·61
1925	14	13	14	7	5	7	41	19	1·39	0·64
20 years	240	249	241	116	113	126	730	355	1·23	0·60
9 years low spot latitude	111	114	100	56	56	56	325	168	1·22	0·63
11 years high spot latitude	129	135	141	60	57	70	405	187	1·24	0·57
9 years of few sunspots	100	112	111	56	57	55	323	168	1·21	0·63
11 years of many sunspots	140	137	130	60	56	71	407	187	1·25	0·57

§ 22. Table XXIV shows the results of an investigation which aimed at clearing up this point. It was anticipated that the marked excess in the character figures in days from 4 to 6 subsequent to selected Q days must arise from a deficiency of Q days, and it was possible it might arise from an excess of D days. Accordingly, summation was made of the D and Q days which were from 4 to 6 days subsequent to the selected

Q days of the 20 years. Results are given for days $n + 4$, $n + 5$ and $n + 6$ separately, and for the three combined, also the ratios which the latter sum bears to the number of occurrences on the basis of pure chance. Results are given for individual years, and for the usual groups of years.

The occurrences to be expected on pure chance in any individual year were roughly 9·85 for each of the three days, or 29·6 for the three combined. On the 20 years this would give approximately 197 for each of the three days and 592 for the three combined.

In the 20 years as a whole the three days $n + 4$ to $n + 6$ show a deficiency of 40 per cent. in the Q days, and an excess of 23 per cent. in the D days. Also, on the whole, day $n + 5$ supplies the largest number of D days and the smallest number of Q days, but the differences between the three days are not large. A deficiency in the Q days, though not so large a deficiency, was quite according to expectation, but the large excess in the number of D days was not. It is not much less than the excess from the three central days of the 54-day pulse. It suggests a very decided tendency for a magnetic storm to follow a few days after a calm.

There seems to be little, if any, real difference between the different classes of years in Table XXIV, but there are conspicuous differences between individual years. It is rather remarkable that the excess of D days, especially in days $n + 5$ and $n + 6$, is specially conspicuous in 1922, the year in which the 27-day interval was best developed. Also, of the two years which show a deficiency in the number of D days, the one in which the deficiency is greatest is 1914, the year in which the 27-day interval is least developed. The two phenomena, however, are by no means always simultaneously prominent. For instance, the D excesses on days $n + 4$ to $n + 6$ are prominent in 1907 and 1917, years in which the 27-day interval is somewhat poorly developed.

It is a little curious that, in investigating the phenomenon which Dr. Deslandres claims to have found, we have come across one which is its exact antithesis.

Conclusions.—The 27-day interval has been found to present itself in disturbance of any size, large or small. No certain difference has been found in the length of the interval, as between years of low and years of high sunspot latitude, or as between years of many and years of few sunspots. No certain departure has been found from 27·0 days in any type of years. There is an apparent tendency for the interval to be greater the larger the primary disturbance. But this is, at least partly, due to the tendency in very large disturbance to rise more quickly than it falls.

The secondary pulse following 27 days after large disturbance is due, in part, to an increase in the proportion of highly disturbed days, but it also owes a good deal to a diminution in the proportion of very quiet days. Similarly, the secondary pulse following 27 days after exceptional magnetic quietness is due, in part, to an increase in the proportion of very quiet days, but it also owes a good deal to a diminution in the proportion of very disturbed days.

The secondary pulse is better developed in years of few than in years of many sunspots, and better in years of low than in years of high spot latitude, but other causes helping or hindering its development seem to exist.

In some years there seems a decided tendency for the international quiet and disturbed days to form members of the same sequence. Supposing magnetic disturbance due to radiation of some kind from the sun, and the solar area effective on any one day to be a narrow zone, then some solar zones must retain their disturbed condition for many solar rotations, whilst others must be alternately much more disturbed than the average zone and much less.

No trace has been found of the existence of any disturbance interval which is a submultiple of 27 days, but there seems in all kinds of years a decided tendency for high disturbance to develop from 4 to 6 days after the occurrence of conspicuous quietness.

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